GAUSS'S LAW

22

22.1. **(a) IDENTIFY** and **SET UP:** $\Phi_E = \int E\cos\phi dA$, where ϕ is the angle between the normal to the sheet $\hat{\boldsymbol{n}}$ and the electric field $\vec{\boldsymbol{E}}$.

EXECUTE: In this problem E and $\cos\phi$ are constant over the surface so

$\Phi_E = E\cos\phi\int dA = E\cos\phi A = (14\ \text{N/C})(\cos 60°)(0.250\ \text{m}^2) = 1.8\ \text{N}\cdot\text{m}^2/\text{C}$.

(b) EVALUATE: Φ_E is independent of the shape of the sheet as long as ϕ and E are constant at all points on the sheet.

(c) EXECUTE: (i) $\Phi_E = E\cos\phi A$. Φ_E is largest for $\phi = 0°$, so $\cos\phi = 1$ and $\Phi_E = EA$.

(ii) Φ_E is smallest for $\phi = 90°$, so $\cos\phi = 0$ and $\Phi_E = 0$.

EVALUATE: Φ_E is 0 when the surface is parallel to the field so no electric field lines pass through the surface.

22.2. **IDENTIFY:** The field is uniform and the surface is flat, so use $\Phi_E = EA\cos\phi$.

SET UP: ϕ is the angle between the normal to the surface and the direction of $\vec{\boldsymbol{E}}$, so $\phi = 70°$.

EXECUTE: $\Phi_E = (75.0\ \text{N/C})(0.400\ \text{m})(0.600\ \text{m})\cos 70° = 6.16\ \text{N}\cdot\text{m}^2/\text{C}$

EVALUATE: If the field were perpendicular to the surface the flux would be $\Phi_E = EA = 18.0\ \text{N}\cdot\text{m}^2/\text{C}$. The flux in this problem is much less than this because only the component of $\vec{\boldsymbol{E}}$ perpendicular to the surface contributes to the flux.

22.3. **IDENTIFY:** The electric flux through an area is defined as the product of the component of the electric field perpendicular to the area times the area.

(a) SET UP: In this case, the electric field is perpendicular to the surface of the sphere, so $\Phi_E = EA = E(4\pi r^2)$.

EXECUTE: Substituting in the numbers gives

$$\Phi_E = (1.25\times10^6\ \text{N/C})4\pi(0.150\,\text{m})^2 = 3.53\times10^5\ \text{N}\cdot\text{m}^2/\text{C}$$

(b) IDENTIFY: We use the electric field due to a point charge.

SET UP: $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2}$

EXECUTE: Solving for q and substituting the numbers gives

$$q = 4\pi\epsilon_0 r^2 E = \frac{1}{9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}(0.150\,\text{m})^2(1.25\times10^6\ \text{N/C}) = 3.13\times10^{-6}\ \text{C}$$

EVALUATE: The flux would be the same no matter how large the sphere, since the area is proportional to r^2 while the electric field is proportional to $1/r^2$.

22.4. **IDENTIFY:** Use Eq.(22.3) to calculate the flux for each surface. Use Eq.(22.8) to calculate the total enclosed charge.

SET UP: $\vec{\boldsymbol{E}} = (-5.00\ \text{N/C}\cdot\text{m})x\hat{\boldsymbol{i}} + (3.00\ \text{N/C}\cdot\text{m})z\hat{\boldsymbol{k}}$. The area of each face is L^2, where $L = 0.300\ \text{m}$.

EXECUTE: $\hat{\boldsymbol{n}}_{S_1} = -\hat{\boldsymbol{j}} \Rightarrow \Phi_1 = \vec{\boldsymbol{E}}\cdot\hat{\boldsymbol{n}}_{S_1}A = 0$.

$\hat{\boldsymbol{n}}_{S_2} = +\hat{\boldsymbol{k}} \Rightarrow \Phi_2 = \vec{\boldsymbol{E}}\cdot\hat{\boldsymbol{n}}_{S_2}A = (3.00\ \text{N/C}\cdot\text{m})(0.300\ \text{m})^2 z = (0.27\ (\text{N/C})\cdot\text{m})z$.

$\Phi_2 = (0.27\ (\text{N/C})\text{m})(0.300\ \text{m}) = 0.081\ (\text{N/C})\cdot\text{m}^2$.

$\hat{\boldsymbol{n}}_{S_3} = +\hat{\boldsymbol{j}} \Rightarrow \Phi_3 = \vec{\boldsymbol{E}}\cdot\hat{\boldsymbol{n}}_{S_3}A = 0$.

$\hat{\boldsymbol{n}}_{S_4} = -\hat{\boldsymbol{k}} \Rightarrow \Phi_4 = \vec{\boldsymbol{E}}\cdot\hat{\boldsymbol{n}}_{S_4}A = -(0.27\ (\text{N/C})\cdot\text{m})z = 0$ (since $z = 0$).

$\hat{\boldsymbol{n}}_{S_5} = +\hat{\boldsymbol{i}} \Rightarrow \Phi_5 = \vec{\boldsymbol{E}}\cdot\hat{\boldsymbol{n}}_{S_5}A = (-5.00\ \text{N/C}\cdot\text{m})(0.300\ \text{m})^2 x = -(0.45\ (\text{N/C})\cdot\text{m})x$.

$\Phi_5 = -(0.45\ (\text{N/C})\cdot\text{m})(0.300\ \text{m}) = -(0.135\ (\text{N/C})\cdot\text{m}^2)$.

$\hat{\boldsymbol{n}}_{S_6} = -\hat{\boldsymbol{i}} \Rightarrow \Phi_6 = \vec{\boldsymbol{E}}\cdot\hat{\boldsymbol{n}}_{S_6}A = +(0.45\ (\text{N/C})\cdot\text{m})x = 0$ (since $x = 0$).

(**b**)Total flux: $\Phi = \Phi_2 + \Phi_5 = (0.081 - 0.135)(\text{N/C})\cdot\text{m}^2 = -0.054\ \text{N}\cdot\text{m}^2/\text{C}$. Therefore, $q = \epsilon_0\Phi = -4.78\times10^{-13}$ C.

EVALUATE: Flux is positive when $\vec{E}$ is directed out of the volume and negative when it is directed into the volume.

22.5. **IDENTIFY:** The flux through the curved upper half of the hemisphere is the same as the flux through the flat circle defined by the bottom of the hemisphere because every electric field line that passes through the flat circle also must pass through the curved surface of the hemisphere.

SET UP: The electric field is perpendicular to the flat circle, so the flux is simply the product of E and the area of the flat circle of radius r.

EXECUTE: $\Phi_E = EA = E(\pi r^2) = \pi r^2 E$

EVALUATE: The flux would be the same if the hemisphere were replaced by any other surface bounded by the flat circle.

22.6. **IDENTIFY:** Use Eq.(22.3) to calculate the flux for each surface.

SET UP: $\Phi = \vec{E}\cdot\vec{A} = EA\cos\phi$ where $\vec{A} = A\hat{n}$.

EXECUTE: (**a**) $\hat{n}_{S_1} = -\hat{j}$ (left). $\Phi_{S_1} = -(4\times10^3\ \text{N/C})(0.10\ \text{m})^2\cos(90° - 36.9°) = -24\ \text{N}\cdot\text{m}^2/\text{C}$.

$\hat{n}_{S_2} = +\hat{k}$ (top). $\Phi_{S_2} = -(4\times10^3\ \text{N/C})(0.10\ \text{m})^2\cos 90° = 0$.

$\hat{n}_{S_3} = +\hat{j}$ (right). $\Phi_{S_3} = +(4\times10^3\ \text{N/C})(0.10\ \text{m})^2\cos(90° - 36.9°) = +24\ \text{N}\cdot\text{m}^2/\text{C}$.

$\hat{n}_{S_4} = -\hat{k}$ (bottom). $\Phi_{S_4} = (4\times10^3\ \text{N/C})(0.10\ \text{m})^2\cos 90° = 0$.

$\hat{n}_{S_5} = +\hat{i}$ (front). $\Phi_{S_5} = +(4\times10^3\ \text{N/C})(0.10\ \text{m})^2\cos 36.9° = 32\ \text{N}\cdot\text{m}^2/\text{C}$.

$\hat{n}_{S_6} = -\hat{i}$ (back). $\Phi_{S_6} = -(4\times10^3\ \text{N/C})(0.10\ \text{m})^2\cos 36.9° = -32\ \text{N}\cdot\text{m}^2/\text{C}$.

EVALUATE: (**b**) The total flux through the cube must be zero; any flux entering the cube must also leave it, since the field is uniform. Our calculation gives the result; the sum of the fluxes calculated in part (a) is zero.

22.7. (**a**) **IDENTIFY:** Use Eq.(22.5) to calculate the flux through the surface of the cylinder.

SET UP: The line of charge and the cylinder are sketched in Figure 22.7.

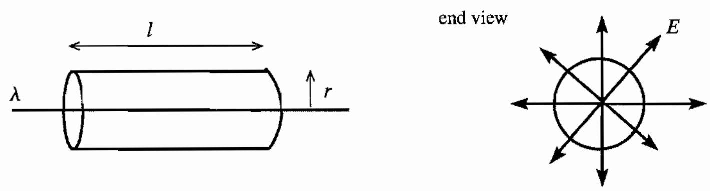


Figure 22.7

EXECUTE: The area of the curved part of the cylinder is $A = 2\pi rl$.

The electric field is parallel to the end caps of the cylinder, so $\vec{E}\cdot\vec{A} = 0$ for the ends and the flux through the cylinder end caps is zero.

The electric field is normal to the curved surface of the cylinder and has the same magnitude $E = \lambda/2\pi\epsilon_0 r$ at all points on this surface. Thus $\phi = 0°$ and

$$\Phi_E = EA\cos\phi = EA = (\lambda/2\pi\epsilon_0 r)(2\pi rl) = \frac{\lambda l}{\epsilon_0} = \frac{(6.00\times10^{-6}\ \text{C/m})(0.400\ \text{m})}{8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 2.71\times10^5\ \text{N}\cdot\text{m}^2/\text{C}$$

(**b**) In the calculation in part (a) the radius r of the cylinder divided out, so the flux remains the same, $\Phi_E = 2.71\times10^5\ \text{N}\cdot\text{m}^2/\text{C}$.

(**c**) $\Phi_E = \dfrac{\lambda l}{\epsilon_0} = \dfrac{(6.00\times10^{-6}\ \text{C/m})(0.800\ \text{m})}{8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 5.42\times10^5\ \text{N}\cdot\text{m}^2/\text{C}$ (twice the flux calculated in parts (b) and (c)).

EVALUATE: The flux depends on the number of field lines that pass through the surface of the cylinder.

22.8. **IDENTIFY:** Apply Gauss's law to each surface.

SET UP: Q_{encl} is the algebraic sum of the charges enclosed by each surface. Flux out of the volume is positive and flux into the enclosed volume is negative.

EXECUTE: (**a**) $\Phi_{S_1} = q_1/\epsilon_0 = (4.00\times10^{-9}\ \text{C})/\epsilon_0 = 452\ \text{N}\cdot\text{m}^2/\text{C}$.

(**b**) $\Phi_{S_2} = q_2/\epsilon_0 = (-7.80\times10^{-9}\ \text{C})/\epsilon_0 = -881\ \text{N}\cdot\text{m}^2/\text{C}$.

(**c**) $\Phi_{S_3} = (q_1 + q_2)/\epsilon_0 = ((4.00 - 7.80)\times10^{-9}\ \text{C})/\epsilon_0 = -429\ \text{N}\cdot\text{m}^2/\text{C}$.

(**d**) $\Phi_{S_4} = (q_1 + q_3)/\epsilon_0 = ((4.00 + 2.40)\times10^{-9}\ \text{C})/\epsilon_0 = 723\ \text{N}\cdot\text{m}^2/\text{C}$.

(**e**) $\Phi_{S_5} = (q_1 + q_2 + q_3)/\epsilon_0 = ((4.00 - 7.80 + 2.40)\times10^{-9}\ \text{C})/\epsilon_0 = -158\ \text{N}\cdot\text{m}^2/\text{C}$.

EVALUATE: **(f)** All that matters for Gauss's law is the total amount of charge enclosed by the surface, not its distribution within the surface.

22.9. **IDENTIFY:** Apply the results in Example 21.10 for the field of a spherical shell of charge.

SET UP: Example 22.10 shows that $E = 0$ inside a uniform spherical shell and that $E = k\dfrac{|q|}{r^2}$ outside the shell.

EXECUTE: **(a)** $E = 0$

(b) $r = 0.060\ \text{m}$ and $E = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{15.0\times10^{-6}\ \text{C}}{(0.060\ \text{m})^2} = 3.75\times10^7\ \text{N/C}$

(c) $r = 0.110\ \text{m}$ and $E = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{15.0\times10^{-6}\ \text{C}}{(0.110\ \text{m})^2} = 1.11\times10^7\ \text{N/C}$

EVALUATE: Outside the shell the electric field is the same as if all the charge were concentrated at the center of the shell. But inside the shell the field is not the same as for a point charge at the center of the shell, inside the shell the electric field is zero.

22.10. **IDENTIFY:** Apply Gauss's law to the spherical surface.

SET UP: Q_{encl} is the algebraic sum of the charges enclosed by the sphere.

EXECUTE: **(a)** No charge enclosed so $\Phi = 0$.

(b) $\Phi = \dfrac{q_2}{\epsilon_0} = \dfrac{-6.00\times10^{-9}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = -678\ \text{N}\cdot\text{m}^2/\text{C}.$

(c) $\Phi = \dfrac{q_1+q_2}{\epsilon_0} = \dfrac{(4.00-6.00)\times10^{-9}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = -226\ \text{N}\cdot\text{m}^2/\text{C}.$

EVALUATE: Negative flux corresponds to flux directed into the enclosed volume. The net flux depends only on the net charge enclosed by the surface and is not affected by any charges outside the enclosed volume.

22.11. **IDENTIFY:** Apply Gauss's law.

SET UP: In each case consider a small Gaussian surface in the region of interest.

EXECUTE: **(a)** Since $\vec{E}$ is uniform, the flux through a closed surface must be zero. That is:

$\Phi = \oint \vec{E}\cdot d\vec{A} = \dfrac{q}{\epsilon_0} = \dfrac{1}{\epsilon_0}\int \rho dV = 0 \Rightarrow \int \rho dV = 0.$ But because we can choose any volume we want, ρ must be zero if the integral equals zero.

(b) If there is no charge in a region of space, that does NOT mean that the electric field is uniform. Consider a closed volume close to, but not including, a point charge. The field diverges there, but there is no charge in that region.

EVALUATE: The electric field within a region can depend on charges located outside the region. But the flux through a closed surface depends only on the net charge contained within that surface.

22.12. **IDENTIFY:** Apply Gauss's law.

SET UP: Use a small Gaussian surface located in the region of question.

EXECUTE: **(a)** If $\rho > 0$ and uniform, then q inside any closed surface is greater than zero. This implies $\Phi > 0$, so $\oint \vec{E}\cdot d\vec{A} > 0$ and so the electric field cannot be uniform. That is, since an arbitrary surface of our choice encloses a non-zero amount of charge, E must depend on position.

(b) However, inside a small bubble of zero charge density within the material with density ρ, the field can be uniform. All that is important is that there be zero flux through the surface of the bubble (since it encloses no charge). (See Problem 22.61.)

EVALUATE: In a region of uniform field, the flux through any closed surface is zero.

22.13. **(a) IDENTIFY** and **SET UP:** It is rather difficult to calculate the flux directly from $\Phi = \oint \vec{E}\cdot d\vec{A}$ since the magnitude of $\vec{E}$ and its angle with $d\vec{A}$ varies over the surface of the cube. A much easier approach is to use Gauss's law to calculate the total flux through the cube. Let the cube be the Gaussian surface. The charge enclosed is the point charge.

EXECUTE: $\Phi_E = Q_{\text{encl}}/\epsilon_0 = \dfrac{9.60\times10^{-6}\ \text{C}}{8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 1.084\times10^6\ \text{N}\cdot\text{m}^2/\text{C}.$

By symmetry the flux is the same through each of the six faces, so the flux through one face is $\frac{1}{6}(1.084\times10^6\ \text{N}\cdot\text{m}^2/\text{C}) = 1.81\times10^5\ \text{N}\cdot\text{m}^2/\text{C}.$

(b) EVALUATE: In part (a) the size of the cube did not enter into the calculations. The flux through one face depends only on the amount of charge at the center of the cube. So the answer to (a) would not change if the size of the cube were changed.

22.14. **IDENTIFY:** Apply the results of Examples 22.9 and 22.10.

SET UP: $E = k\dfrac{|q|}{r^2}$ outside the sphere. A proton has charge $+e$.

EXECUTE: **(a)** $E = k\dfrac{|q|}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{92(1.60\times10^{-19}\ \text{C})}{(7.4\times10^{-15}\ \text{m})^2} = 2.4\times10^{21}\ \text{N/C}$

(b) For $r = 1.0\times10^{-10}\ \text{m}$, $E = (2.4\times10^{21}\ \text{N/C})\left(\dfrac{7.4\times10^{-15}\ \text{m}}{1.0\times10^{-10}\ \text{m}}\right)^2 = 1.3\times10^{13}\ \text{N/C}$

(c) $E = 0$, inside a spherical shell.

EVALUATE: The electric field in an atom is very large.

22.15. **IDENTIFY:** The electric fields are produced by point charges.

SET UP: We use Coulomb's law, $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2}$, to calculate the electric fields.

EXECUTE: **(a)** $E = \left(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\dfrac{5.00\times10^{-6}\ \text{C}}{(1.00\ \text{m})^2} = 4.50\times10^4\ \text{N/C}$

(b) $E = \left(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\dfrac{5.00\times10^{-6}\ \text{C}}{(7.00\ \text{m})^2} = 9.18\times10^2\ \text{N/C}$

(c) Every field line that enters the sphere on one side leaves it on the other side, so the net flux through the surface is zero.

EVALUATE: The flux would be zero no matter what shape the surface had, providing that no charge was inside the surface.

22.16. **IDENTIFY:** Apply the results of Example 22.5.

SET UP: At a point 0.100 m outside the surface, $r = 0.550\ \text{m}$.

EXECUTE: **(a)** $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2} = \dfrac{1}{4\pi\epsilon_0}\dfrac{(2.50\times10^{-10}\ \text{C})}{(0.550\ \text{m})^2} = 7.44\ \text{N/C}.$

(b) $E = 0$ inside of a conductor or else free charges would move under the influence of forces, violating our electrostatic assumptions (i.e., that charges aren't moving).

EVALUATE: Outside the sphere its electric field is the same as would be produced by a point charge at its center, with the same charge.

22.17. **IDENTIFY:** The electric field required to produce a spark 6 in. long is 6 times as strong as the field needed to produce a spark 1 in. long.

SET UP: By Gauss's law, $q = \epsilon_0 EA$ and the electric field is the same as for a point-charge, $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2}$.

EXECUTE: **(a)** The electric field for 6-inch sparks is $E = 6\times2.00\times10^4\ \text{N/C} = 1.20\times10^5\ \text{N/C}$

The charge to produce this field is

$q = \epsilon_0 EA = \epsilon_0 E(4\pi r^2) = (8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(1.20\times10^5\ \text{N/C})(4\pi)(0.15\text{m})^2 = 3.00\times10^{-7}\ \text{C}.$

(b) Using Coulomb's law gives $E = (9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{3.00\times10^{-7}\ \text{C}}{(0.150\ \text{m})^2} = 1.20\times10^5\ \text{N/C}.$

EVALUATE: It takes only about $0.3\ \mu\text{C}$ to produce a field this strong.

22.18. **IDENTIFY:** According to Exercise 21.32, the Earth's electric field points towards its center. Since Mars's electric field is similar to that of Earth, we assume it points toward the center of Mars. Therefore the charge on Mars must be negative. We use Gauss's law to relate the electric flux to the charge causing it.

SET UP: Gauss's law is $\Phi_E = \dfrac{q}{\epsilon_0}$ and the electric flux is $\Phi_E = EA$.

EXECUTE: **(a)** Solving Gauss's law for q, putting in the numbers, and recalling that q is negative, gives

$q = -\epsilon_0\Phi_E = -(3.63\times10^{16}\ \text{N}\cdot\text{m}^2/\text{C})(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2) = -3.21\times10^5\ \text{C}.$

(b) Use the definition of electric flux to find the electric field. The area to use is the surface area of Mars.

$$E = \frac{\Phi_E}{A} = \frac{3.63\times10^{16}\ \text{N}\cdot\text{m}^2/\text{C}}{4\pi(3.40\times10^6\ \text{m})^2} = 2.50\times10^2\ \text{N/C}$$

(c) The surface charge density on Mars is therefore $\sigma = \dfrac{q}{A_{\text{Mars}}} = \dfrac{-3.21\times10^5\ \text{C}}{4\pi(3.40\times10^6\ \text{m})^2} = -2.21\times10^{-9}\ \text{C/m}^2$

EVALUATE: Even though the charge on Mars is very large, it is spread over a large area, giving a small surface charge density.

22.19. **IDENTIFY and SET UP:** Example 22.5 derived that the electric field just outside the surface of a spherical conductor that has net charge q is $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{R^2}$. Calculate q and from this the number of excess electrons.

EXECUTE: $q = \dfrac{R^2 E}{(1/4\pi\epsilon_0)} = \dfrac{(0.160\ \text{m})^2(1150\ \text{N/C})}{8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 3.275\times10^{-9}\ \text{C}.$

Each electron has a charge of magnitude $e = 1.602\times10^{-19}$ C, so the number of excess electrons needed is

$\dfrac{3.275\times10^{-9}\ \text{C}}{1.602\times10^{-19}\ \text{C}} = 2.04\times10^{10}.$

EVALUATE: The result we obtained for q is a typical value for the charge of an object. Such net charges correspond to a large number of excess electrons since the charge of each electron is very small.

22.20. **IDENTIFY:** Apply Gauss's law.

SET UP: Draw a cylindrical Gaussian surface with the line of charge as its axis. The cylinder has radius 0.400 m and is 0.0200 m long. The electric field is then 840 N/C at every point on the cylindrical surface and is directed perpendicular to the surface.

EXECUTE: $\oint \vec{E}\cdot d\vec{A} = EA_{\text{cylinder}} = E(2\pi rL) = (840\ \text{N/C})(2\pi)(0.400\ \text{m})(0.0200\ \text{m}) = 42.2\ \text{N}\cdot\text{m}^2/\text{C}.$

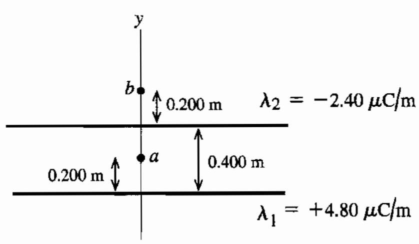
The field is parallel to the end caps of the cylinder, so for them $\oint \vec{E}\cdot d\vec{A} = 0.$ From Gauss's law,

$q = \epsilon_0\Phi_E = (8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(42.2\ \text{N}\cdot\text{m}^2/\text{C}) = 3.74\times10^{-10}\ \text{C}.$

EVALUATE: We could have applied the result in Example 22.6 and solved for λ. Then $q = \lambda L.$

22.21. **IDENTIFY:** Add the vector electric fields due to each line of charge. $E(r)$ for a line of charge is given by Example 22.6 and is directed toward a negative line of chage and away from a positive line.

SET UP: The two lines of charge are shown in Figure 22.21.



$$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}$$

Figure 22.21

EXECUTE: **(a)** At point a, $\vec{E}_1$ and $\vec{E}_2$ are in the $+y$-direction (toward negative charge, away from positive charge).

$E_1 = (1/2\pi\epsilon_0)\left[(4.80\times10^{-6}\ \text{C/m})/(0.200\ \text{m})\right] = 4.314\times10^5\ \text{N/C}$

$E_2 = (1/2\pi\epsilon_0)\left[(2.40\times10^{-6}\ \text{C/m})/(0.200\ \text{m})\right] = 2.157\times10^5\ \text{N/C}$

$E = E_1 + E_2 = 6.47\times10^5$ N/C, in the y-direction.

(b) At point b, $\vec{E}_1$ is in the $+y$-direction and $\vec{E}_2$ is in the $-y$-direction.

$E_1 = (1/2\pi\epsilon_0)\left[(4.80\times10^{-6}\ \text{C/m})/(0.600\ \text{m})\right] = 1.438\times10^5\ \text{N/C}$

$E_2 = (1/2\pi\epsilon_0)\left[(2.40\times10^{-6}\ \text{C/m})/(0.200\ \text{m})\right] = 2.157\times10^5\ \text{N/C}$

$E = E_2 - E_1 = 7.2\times10^4$ N/C, in the $-y$-direction.

EVALUATION: At point a the two fields are in the same direction and the magnitudes add. At point b the two fields are in opposite directions and the magnitudes subtract.

22.22. **IDENTIFY:** Apply the results of Examples 22.5, 22.6 and 22.7.

SET UP: Gauss's law can be used to show that the field outside a long conducting cylinder is the same as for a line of charge along the axis of the cylinder.

EXECUTE: **(a)** For points outside a uniform spherical charge distribution, all the charge can be considered to be concentrated at the center of the sphere. The field outside the sphere is thus inversely proportional to the square of the distance from the center. In this case,

$$E = (480\ \text{N/C})\left(\frac{0.200\ \text{cm}}{0.600\ \text{cm}}\right)^2 = 53\ \text{N/C}$$

(b) For points outside a long cylindrically symmetrical charge distribution, the field is identical to that of a long line of charge: $E = \dfrac{\lambda}{2\pi\epsilon_0 r}$, that is, inversely proportional to the distance from the axis of the cylinder. In this case

$$E = (480 \text{ N/C})\left(\frac{0.200 \text{ cm}}{0.600 \text{ cm}}\right) = 160 \text{ N/C}$$

(c) The field of an infinite sheet of charge is $E = \sigma/2\epsilon_0$; i.e., it is independent of the distance from the sheet. Thus in this case $E = 480$ N/C.

EVALUATE: For each of these three distributions of charge the electric field has a different dependence on distance.

22.23. **IDENTIFY:** The electric field inside the conductor is zero, and all of its initial charge lies on its outer surface. The introduction of charge into the cavity induces charge onto the surface of the cavity, which induces an equal but opposite charge on the outer surface of the conductor. The net charge on the outer surface of the conductor is the sum of the positive charge initially there and the additional negative charge due to the introduction of the negative charge into the cavity.

(a) SET UP: First find the initial positive charge on the outer surface of the conductor using $q_i = \sigma A$, where A is the area of its outer surface. Then find the net charge on the surface after the negative charge has been introduced into the cavity. Finally use the definition of surface charge density.

EXECUTE: The original positive charge on the outer surface is

$$q_i = \sigma A = \sigma(4\pi r^2) = (6.37\times10^{-6} \text{ C/m}^2)4\pi(0.250 \text{ m})^2 = 5.00\times10^{-6} \text{ C/m}^2$$

After the introduction of $-0.500\ \mu$C into the cavity, the outer charge is now

$$5.00\ \mu\text{C} - 0.500\ \mu\text{C} = 4.50\ \mu\text{C}$$

The surface charge density is now $\sigma = \dfrac{q}{A} = \dfrac{q}{4\pi r^2} = \dfrac{4.50\times10^{-6} \text{ C}}{4\pi(0.250 \text{ m})^2} = 5.73\times10^{-6} \text{ C/m}^2$

(b) SET UP: Using Gauss's law, the electric field is $E = \dfrac{\Phi_E}{A} = \dfrac{q}{\epsilon_0 A} = \dfrac{q}{\epsilon_0 4\pi r^2}$

EXECUTE: Substituting numbers gives

$$E = \frac{4.50\times10^{-6} \text{ C}}{(8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(4\pi)(0.250 \text{ m})^2} = 6.47\times10^5 \text{ N/C}.$$

(c) SET UP: We use Gauss's law again to find the flux. $\Phi_E = \dfrac{q}{\epsilon_0}$.

EXECUTE: Substituting numbers gives

$$\Phi_E = \frac{-0.500\times10^{-6} \text{ C}}{8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2} = -5.65\times10^4 \text{ N}\cdot\text{m}^2/\text{C}^2.$$

EVALUATE: The excess charge on the conductor is still $+5.00\ \mu$C, as it originally was. The introduction of the $-0.500\ \mu$C inside the cavity merely induced equal but opposite charges (for a net of zero) on the surfaces of the conductor.

22.24. **IDENTIFY:** We apply Gauss's law, taking the Gaussian surface beyond the cavity but inside the solid.

SET UP: Because of the symmetry of the charge, Gauss's law gives us $E = \dfrac{q_{\text{total}}}{\epsilon_0 A}$, where A is the surface area of a sphere of radius $R = 9.50$ cm centered on the point-charge, and q_{total} is the total charge contained within that sphere. This charge is the sum of the $-2.00\ \mu$C point charge at the center of the cavity plus the charge within the solid between $r = 6.50$ cm and $R = 9.50$ cm. The charge within the solid is $q_{\text{solid}} = \rho V = \rho([4/3]\pi R^3 - [4/3]\pi r^3) = ([4\pi/3]\rho)(R^3 - r^3)$

EXECUTE: First find the charge within the solid between $r = 6.50$ cm and $R = 9.50$ cm:

$$q_{\text{solid}} = \frac{4\pi}{3}(7.35\times10^{-4} \text{ C/m}^3)\left[(0.0950 \text{ m})^3 - (0.0650 \text{ m})^3\right] = 1.794\times10^{-6} \text{ C},$$

Now find the total charge within the Gaussian surface:

$$q_{\text{total}} = q_{\text{solid}} + q_{\text{point}} = -2.00\ \mu\text{C} + 1.794\ \mu\text{C} = -0.2059\ \mu\text{C}$$

Now find the magnitude of the electric field from Gauss's law:

$$E = \frac{q}{\epsilon_0 A} = \frac{q}{\epsilon_0 4\pi r^2} = \frac{0.2059\times10^{-6} \text{ C}}{(8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(4\pi)(0.0950 \text{ m})^2} = 2.05\times10^5 \text{ N/C}.$$

The fact that the charge is negative means that the electric field points radially inward.

EVALUATE: Because of the uniformity of the charge distribution, the charge beyond 9.50 cm does not contribute to the electric field.

22.25. **IDENTIFY:** The magnitude of the electric field is constant at any given distance from the center because the charge density is uniform inside the sphere. We can use Gauss's law to relate the field to the charge causing it.

(a) SET UP: Gauss's law tells us that $EA = \frac{q}{\epsilon_0}$, and the charge density is given by $\rho = \frac{q}{V} = \frac{q}{(4/3)\pi R^3}$.

EXECUTE: Solving for q and substituting numbers gives
$q = EA\epsilon_0 = E(4\pi r^2)\epsilon_0 = (1750\text{ N/C})(4\pi)(0.500\text{ m})^2(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2) = 4.866\times10^{-8}\text{ C}$. Using the formula for charge density we get $\rho = \frac{q}{V} = \frac{q}{(4/3)\pi R^3} = \frac{4.866\times10^{-8}\text{ C}}{(4/3)\pi(0.355\text{ m})^3} = 2.60\times10^{-7}\text{ C/m}^3$.

(b) SET UP: Take a Gaussian surface of radius $r = 0.200$ m, concentric with the insulating sphere. The charge enclosed within this surface is $q_{\text{encl}} = \rho V = \rho\left(\frac{4}{3}\pi r^3\right)$, and we can treat this charge as a point-charge, using Coulomb's law $E = \frac{1}{4\pi\epsilon_0}\frac{q_{\text{encl}}}{r^2}$. The charge beyond $r = 0.200$ m makes no contribution to the electric field.

EXECUTE: First find the enclosed charge:

$$q_{\text{encl}} = \rho\left(\frac{4}{3}\pi r^3\right) = \left(2.60\times10^{-7}\text{ C/m}^3\right)\left[\frac{4}{3}\pi\left(0.200\text{m}\right)^3\right] = 8.70\times10^{-9}\text{ C}$$

Now treat this charge as a point-charge and use Coulomb's law to find the field:

$$E = \left(9.00\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\frac{8.70\times10^{-9}\text{ C}}{\left(0.200\text{ m}\right)^2} = 1.96\times10^3\text{ N/C}$$

EVALUATE: Outside this sphere, it behaves like a point-charge located at its center. Inside of it, at a distance r from the center, the field is due only to the charge between the center and r.

22.26. **IDENTIFY:** Apply Gauss's law and conservation of charge.

SET UP: Use a Gaussian surface that lies wholly within the conducting material.

EXECUTE: **(a)** Positive charge is attracted to the inner surface of the conductor by the charge in the cavity. Its magnitude is the same as the cavity charge: $q_{inner} = +6.00\text{ nC}$, since $E = 0$ inside a conductor and a Gaussian surface that lies wholly within the conductor must enclose zero net charge.

(b) On the outer surface the charge is a combination of the net charge on the conductor and the charge "left behind" when the $+6.00\text{ nC}$ moved to the inner surface:

$$q_{\text{tot}} = q_{\text{inner}} + q_{\text{outer}} \Rightarrow q_{\text{outer}} = q_{\text{tot}} - q_{\text{inner}} = 5.00\text{ nC} - 6.00\text{ nC} = -1.00\text{ nC}.$$

EVALUATE: The electric field outside the conductor is due to the charge on its surface.

22.27. **IDENTIFY:** Apply Gauss's law to each surface.

SET UP: The field is zero within the plates. By symmetry the field is perpendicular to the plates outside the plates and can depend only on the distance from the plates. Flux into the enclosed volume is positive.

EXECUTE: S_2 and S_3 enclose no charge, so the flux is zero, and electric field outside the plates is zero. Between the plates, S_4 shows that $-EA = -q/\epsilon_0 = -\sigma A/\epsilon_0$ and $E = \sigma/\epsilon_0$.

EVALUATE: Our result for the field between the plates agrees with the result stated in Example 22.8.

22.28. **IDENTIFY:** Close to a finite sheet the field is the same as for an infinite sheet. Very far from a finite sheet the field is that of a point charge.

SET UP: For an infinite sheet, $E = \frac{\sigma}{2\epsilon_0}$. For a point charge, $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$.

EXECUTE: **(a)** At a distance of 0.1 mm from the center, the sheet appears "infinite," so

$$E \approx \frac{q}{2\epsilon_0 A} = \frac{7.50\times10^{-9}\text{ C}}{2\epsilon_0(0.800\text{ m})^2} = 662\text{ N/C}.$$

(b) At a distance of 100 m from the center, the sheet looks like a point, so:

$$E \approx \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{(7.50\times10^{-9}\text{ C})}{(100\text{ m})^2} = 6.75\times10^{-3}\text{ N/C}.$$

(c) There would be no difference if the sheet was a conductor. The charge would automatically spread out evenly over both faces, giving it half the charge density on either face as the insulator but the same electric field. Far away, they both look like points with the same charge.

EVALUATE: The sheet can be treated as infinite at points where the distance to the sheet is much less than the distance to the edge of the sheet. The sheet can be treated as a point charge at points for which the distance to the sheet is much greater than the dimensions of the sheet.

22.29. **IDENTIFY:** Apply Gauss's law to a Gaussian surface and calculate E.

(a) SET UP: Consider the charge on a length l of the cylinder. This can be expressed as $q = \lambda l$. But since the surface area is $2\pi Rl$ it can also be expressed as $q = \sigma 2\pi Rl$. These two expressions must be equal, so $\lambda l = \sigma 2\pi Rl$ and $\lambda = 2\pi R\sigma$.

(b) Apply Gauss's law to a Gaussian surface that is a cylinder of length l, radius r, and whose axis coincides with the axis of the charge distribution, as shown in Figure 22.29.

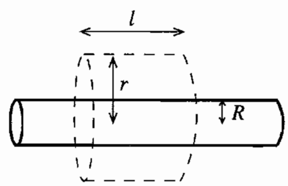


Figure 22.29

EXECUTE:
$Q_{\text{encl}} = \sigma(2\pi Rl)$
$\Phi_E = 2\pi rlE$

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $2\pi rlE = \dfrac{\sigma(2\pi Rl)}{\epsilon_0}$

$$E = \frac{\sigma R}{\epsilon_0 r}$$

(c) EVALUATE: Example 22.6 shows that the electric field of an infinite line of charge is $E = \lambda / 2\pi\epsilon_0 r$. $\sigma = \dfrac{\lambda}{2\pi R}$, so $E = \dfrac{\sigma R}{\epsilon_0 r} = \dfrac{R}{\epsilon_0 r}\left(\dfrac{\lambda}{2\pi R}\right) = \dfrac{\lambda}{2\pi\epsilon_0 r}$, the same as for an infinite line of charge that is along the axis of the cylinder.

22.30. **IDENTIFY:** The net electric field is the vector sum of the fields due to each of the four sheets of charge.

SET UP: The electric field of a large sheet of charge is $E = \sigma / 2\epsilon_0$. The field is directed away from a positive sheet and toward a negative sheet.

EXECUTE: **(a)** At A: $E_A = \dfrac{|\sigma_2|}{2\epsilon_0} + \dfrac{|\sigma_3|}{2\epsilon_0} + \dfrac{|\sigma_4|}{2\epsilon_0} - \dfrac{|\sigma_1|}{2\epsilon_0} = \dfrac{1}{2\epsilon_0}(|\sigma_2| + |\sigma_3| + |\sigma_4| - |\sigma_1|)$.

$E_A = \dfrac{1}{2\epsilon_0}(5\ \mu\text{C/m}^2 + 2\ \mu\text{C/m}^2 + 4\ \mu\text{C/m}^2 - 6\ \mu\text{C/m}^2) = 2.82 \times 10^5$ N/C to the left.

(b) $E_B = \dfrac{|\sigma_1|}{2\epsilon_0} + \dfrac{|\sigma_3|}{2\epsilon_0} + \dfrac{|\sigma_4|}{2\epsilon_0} - \dfrac{|\sigma_2|}{2\epsilon_0} = \dfrac{1}{2\epsilon_0}(|\sigma_1| + |\sigma_3| + |\sigma_4| - |\sigma_2|)$.

$E_B = \dfrac{1}{2\epsilon_0}(6\ \mu\text{C/m}^2 + 2\ \mu\text{C/m}^2 + 4\ \mu\text{C/m}^2 - 5\ \mu\text{C/m}^2) = 3.95 \times 10^5$ N/C to the left.

(c) $E_C = \dfrac{|\sigma_4|}{2\epsilon_0} + \dfrac{|\sigma_1|}{2\epsilon_0} - \dfrac{|\sigma_2|}{2\epsilon_0} - \dfrac{|\sigma_3|}{2\epsilon_0} = \dfrac{1}{2\epsilon_0}(|\sigma_2| + |\sigma_3| - |\sigma_4| - |\sigma_1|)$.

$E_C = \dfrac{1}{2\epsilon_0}(4\ \mu\text{C/m}^2 + 6\ \mu\text{C/m}^2 - 5\ \mu\text{C/m}^2 - 2\ \mu\text{C/m}^2) = 1.69 \times 10^5$ N/C to the left

EVALUATE: The field at C is not zero. The pieces of plastic are not conductors.

22.31. **IDENTIFY:** Apply Gauss's law and conservation of charge.

SET UP: $E = 0$ in a conducting material.

EXECUTE: **(a)** Gauss's law says $+Q$ on inner surface, so $E = 0$ inside metal.

(b) The outside surface of the sphere is grounded, so no excess charge.

(c) Consider a Gaussian sphere with the $-Q$ charge at its center and radius less than the inner radius of the metal. This sphere encloses net charge $-Q$ so there is an electric field flux through it; there is electric field in the cavity.

(d) In an electrostatic situation $E = 0$ inside a conductor. A Gaussian sphere with the $-Q$ charge at its center and radius greater than the outer radius of the metal encloses zero net charge (the $-Q$ charge and the $+Q$ on the inner surface of the metal) so there is no flux through it and $E = 0$ outside the metal.

(e) No, $E = 0$ there. Yes, the charge has been shielded by the grounded conductor. There is nothing like positive and negative mass (the gravity force is always attractive), so this cannot be done for gravity.

EVALUATE: Field lines within the cavity terminate on the charges induced on the inner surface.

22.32. **IDENTIFY** and **SET UP:** Eq.(22.3) to calculate the flux. Identify the direction of the normal unit vector $\hat{\boldsymbol{n}}$ for each surface.

EXECUTE: **(a)** $\vec{E} = -B\hat{i} + C\hat{j} - D\hat{k}$; $A = L^2$

face S_1: $\hat{n} = -\hat{j}$

$\Phi_E = \vec{E}\cdot\vec{A} = \vec{E}\cdot(A\hat{n}) = (-B\hat{i} + C\hat{j} - D\hat{k})\cdot(-A\hat{j}) = -CL^2.$

face S_2: $\hat{n} = +\hat{k}$

$\Phi_E = \vec{E}\cdot\vec{A} = \vec{E}\cdot(A\hat{n}) = (-B\hat{i} + C\hat{j} - D\hat{k})\cdot(A\hat{k}) = -DL^2.$

face S_3: $\hat{n} = +\hat{j}$

$\Phi_E = \vec{E}\cdot\vec{A} = \vec{E}\cdot(A\hat{n}) = (-B\hat{i} + C\hat{j} - D\hat{k})\cdot(A\hat{j}) = +CL^2.$

face S_4: $\hat{n} = -\hat{k}$

$\Phi_E = \vec{E}\cdot\vec{A} = \vec{E}\cdot(A\hat{n}) = (-B\hat{i} + C\hat{j} - D\hat{k})\cdot(-A\hat{k}) = +DL^2.$

face S_5: $\hat{n} = +\hat{i}$

$\Phi_E = \vec{E}\cdot\vec{A} = \vec{E}\cdot(A\hat{n}) = (-B\hat{i} + C\hat{j} - D\hat{k})\cdot(A\hat{i}) = -BL^2.$

face S_6: $\hat{n} = -\hat{i}$

$\Phi_E = \vec{E}\cdot\vec{A} = \vec{E}\cdot(A\hat{n}) = (-B\hat{i} + C\hat{j} - D\hat{k})\cdot(-A\hat{i}) = +BL^2.$

(b) Add the flux through each of the six faces: $\Phi_E = -CL^2 - DL^2 + CL^2 + DL^2 - BL^2 + BL^2 = 0$

The total electric flux through all sides is zero.

EVALUATE: All electric field lines that enter one face of the cube leave through another face. No electric field lines terminate inside the cube and the net flux is zero.

22.33. **IDENTIFY:** Use Eq.(22.3) to calculate the flux through each surface and use Gauss's law to relate the net flux to the enclosed charge.

SET UP: Flux into the enclosed volume is negative and flux out of the volume is positive.

EXECUTE: **(a)** $\Phi = EA = (125\ \text{N/C})(6.0\ \text{m}^2) = 750\ \text{N}\cdot\text{m}^2/\text{C}.$

(b) Since the field is parallel to the surface, $\Phi = 0$.

(c) Choose the Gaussian surface to equal the volume's surface. Then $750\ \text{N}\cdot\text{m}^2/\text{C} - EA = q/\epsilon_0$ and

$E = \dfrac{1}{6.0\ \text{m}^2}(2.40\times10^{-8}\ \text{C}/\epsilon_0 + 750\ \text{N}\cdot\text{m}^2/\text{C}) = 577\ \text{N/C}$, in the positive x-direction. Since $q < 0$ we must have some net flux flowing *in* so the flux is $-|EA|$ on second face.

EVALUATE: **(d)** $q < 0$ but we have E pointing *away* from face I. This is due to an external field that does not affect the flux but affects the value of E.

22.34. **IDENTIFY:** Apply Gauss's law to a cube centered at the origin and with side length $2L$.

SET UP: The total surface area of a cube with side length $2L$ is $6(2L)^2 = 24L^2$.

EXECUTE: **(a)** The square is sketched in Figure 22.34.

(b) Imagine a charge q at the center of a cube of edge length $2L$. Then: $\Phi = q/\epsilon_0$. Here the square is one 24th of the surface area of the imaginary cube, so it intercepts 1/24 of the flux. That is, $\Phi = q/24\epsilon_0$.

EVALUATE: Calculating the flux directly from Eq.(22.5) would involve a complicated integral. Using Gauss's law and symmetry considerations is much simpler.

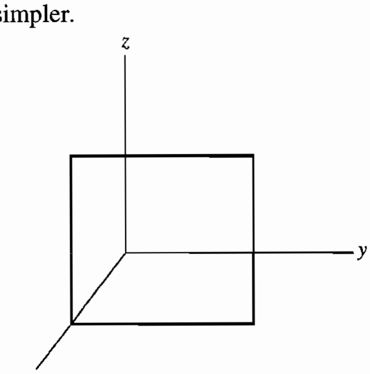


Figure 22.34

22.35. (a) **IDENTIFY:** Find the net flux through the parallelepiped surface and then use that in Gauss's law to find the net charge within. Flux out of the surface is positive and flux into the surface is negative.

SET UP: $\vec{E}_1$ gives flux out of the surface. See Figure 22.35a.

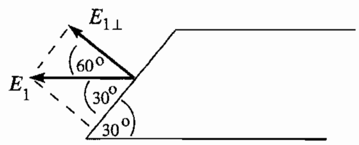


Figure 22.35a

EXECUTE: $\Phi_1 = +E_{1\perp}A$

$A = (0.0600\text{ m})(0.0500\text{ m}) = 3.00\times10^{-3}\text{ m}^2$

$E_{1\perp} = E_1\cos60° = (2.50\times10^4\text{ N/C})\cos60°$

$E_{1\perp} = 1.25\times10^4\text{ N/C}$

$\Phi_{E_1} = +E_{1\perp}A = +(1.25\times10^4\text{ N/C})(3.00\times10^{-3}\text{ m}^2) = 37.5\text{ N}\cdot\text{m}^2/\text{C}$

SET UP: $\vec{E}_2$ gives flux into the surface. See Figure 22.35b.

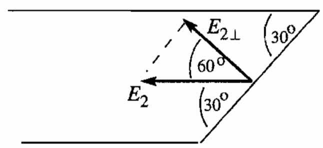


Figure 22.35b

EXECUTE: $\Phi_2 = -E_{2\perp}A$

$A = (0.0600\text{ m})(0.0500\text{ m}) = 3.00\times10^{-3}\text{m}^2$

$E_{2\perp} = E_2\cos60° = (7.00\times10^4\text{ N/C})\cos60°$

$E_{2\perp} = 3.50\times10^4\text{ N/C}$

$\Phi_{E_2} = -E_{2\perp}A = -(3.50\times10^4\text{ N/C})(3.00\times10^{-3}\text{ m}^2) = -105.0\text{ N}\cdot\text{m}^2/\text{C}$

The net flux is $\Phi_E = \Phi_{E_1} + \Phi_{E_2} = +37.5\text{ N}\cdot\text{m}^2/\text{C} - 105.0\text{ N}\cdot\text{m}^2/\text{C} = -67.5\text{ N}\cdot\text{m}^2/\text{C}$.

The net flux is negative (inward), so the net charge enclosed is negative.

Apply Gauss's law: $\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$

$Q_{\text{encl}} = \Phi_E\epsilon_0 = (-67.5\text{ N}\cdot\text{m}^2/\text{C})(8.854\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2) = -5.98\times10^{-10}\text{ C}$.

(b) **EVALUATE:** If there were no charge within the parallelpiped the net flux would be zero. This is not the case, so there is charge inside. The electric field lines that pass out through the surface of the parallelpiped must terminate on charges, so there also must be charges outside the parallelpiped.

22.36. **IDENTIFY:** The α particle feels no force where the net electric field due to the two distributions of charge is zero.

SET UP: The fields can cancel only in the regions A and B shown in Figure 22.36, because only in these two regions are the two fields in opposite directions.

EXECUTE: $E_{\text{line}} = E_{\text{sheet}}$ gives $\dfrac{\lambda}{2\pi\epsilon_0 r} = \dfrac{\sigma}{2\epsilon_0}$ and $r = \lambda/\pi\sigma = \dfrac{50\ \mu\text{C/m}}{\pi(100\ \mu\text{C/m}^2)} = 0.16\text{ m} = 16\text{ cm}$.

The fields cancel 16 cm from the line in regions A and B.

EVALUATE: The result is independent of the distance between the line and the sheet. The electric field of an infinite sheet of charge is uniform, independent of the distance from the sheet.

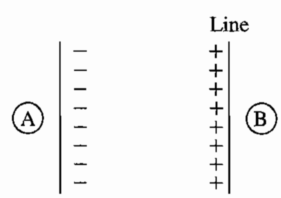


Figure 22.36

22.37. (a) **IDENTIFY:** Apply Gauss's law to a Gaussian cylinder of length l and radius r, where $a < r < b$, and calculate E on the surface of the cylinder.

SET UP: The Gaussian surface is sketched in Figure 22.37a.

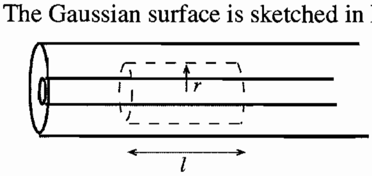


Figure 22.37a

EXECUTE: $\Phi_E = E(2\pi rl)$

$Q_{\text{encl}} = \lambda l$ (the charge on the length l of the inner conductor that is inside the Gaussian surface).

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(2\pi rl) = \dfrac{\lambda l}{\epsilon_0}$

$E = \dfrac{\lambda}{2\pi\epsilon_0 r}$. The enclosed charge is positive so the direction of $\vec{E}$ is radially outward.

(b) SET UP: Apply Gauss's law to a Gaussian cylinder of length l and radius r, where $r > c$, as shown in Figure 22.37b.

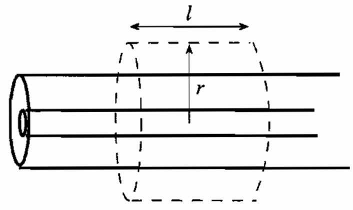


Figure 22.37b

EXECUTE: $\Phi_E = E(2\pi rl)$

$Q_{\text{encl}} = \lambda l$ (the charge on the length l of the inner conductor that is inside the Gaussian surface; the outer conductor carries no net charge).

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(2\pi rl) = \dfrac{\lambda l}{\epsilon_0}$

$E = \dfrac{\lambda}{2\pi\epsilon_0 r}$. The enclosed charge is positive so the direction of $\vec{E}$ is radially outward.

(c) $E = 0$ within a conductor. Thus $E = 0$ for $r < a$;

$E = \dfrac{\lambda}{2\pi\epsilon_0 r}$ for $a < r < b$; $E = 0$ for $b < r < c$;

$E = \dfrac{\lambda}{2\pi\epsilon_0 r}$ for $r > c$. The graph of E versus r is sketched in Figure 22.37c.

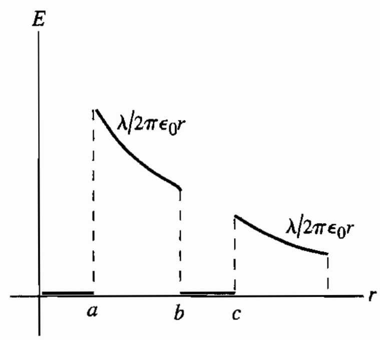


Figure 22.37c

EVALUATE: Inside either conductor $E = 0$. Between the conductors and outside both conductors the electric field is the same as for a line of charge with linear charge density λ lying along the axis of the inner conductor.

(d) IDENTIFY and **SET UP:** <u>inner surface:</u> Apply Gauss's law to a Gaussian cylinder with radius r, where $b < r < c$. We know E on this surface; calculate Q_{encl}.

EXECUTE: This surface lies within the conductor of the outer cylinder, where $E = 0$, so $\Phi_E = 0$. Thus by Gauss's law $Q_{\text{encl}} = 0$. The surface encloses charge λl on the inner conductor, so it must enclose charge $-\lambda l$ on the inner surface of the outer conductor. The charge per unit length on the inner surface of the outer cylinder is $-\lambda$.

<u>outer surface:</u> The outer cylinder carries no net charge. So if there is charge per unit length $-\lambda$ on its inner surface there must be charge per until length $+\lambda$ on the outer surface.

EVALUATE: The electric field lines between the conductors originate on the surface charge on the outer surface of the inner conductor and terminate on the surface charges on the inner surface of the outer conductor. These surface charges are equal in magnitude (per unit length) and opposite in sign. The electric field lines outside the outer conductor originate from the surface charge on the outer surface of the outer conductor.

22.38. **IDENTIFY:** Apply Gauss's law.

SET UP: Use a Gaussian surface that is a cylinder of radius r, length l and that has the line of charge along its axis. The charge on a length l of the line of charge or of the tube is $q = \alpha l$.

EXECUTE: **(a)** (i) For $r < a$, Gauss's law gives $E(2\pi rl) = \dfrac{Q_{\text{encl}}}{\epsilon_0} = \dfrac{\alpha l}{\epsilon_0}$ and $E = \dfrac{\alpha}{2\pi\epsilon_0 r}$.

(ii) The electric field is zero because these points are within the conducting material.

(iii) For $r > b$, Gauss's law gives $E(2\pi rl) = \dfrac{Q_{\text{encl}}}{\epsilon_0} = \dfrac{2\alpha l}{\epsilon_0}$ and $E = \dfrac{\alpha}{\pi\epsilon_0 r}$.

The graph of E versus r is sketched in Figure 22.38.

(b) (i) The Gaussian cylinder with radius r, for $a < r < b$, must enclose zero net charge, so the charge per unit length on the inner surface is $-\alpha$. (ii) Since the net charge per length for the tube is $+\alpha$ and there is $-\alpha$ on the inner surface, the charge per unit length on the outer surface must be $+2\alpha$.

EVALUATE: For $r > b$ the electric field is due to the charge on the outer surface of the tube.

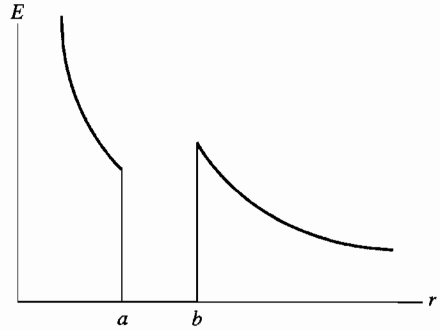


Figure 22.38

22.39. **(a) IDENTIFY:** Use Gauss's law to calculate $E(r)$.

(i) **SET UP:** <u>$r < a$:</u> Apply Gauss's law to a cylindrical Gaussian surface of length l and radius r, where $r < a$, as sketched in Figure 22.39a.

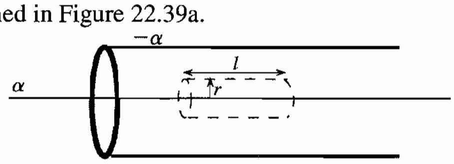


Figure 22.39a

EXECUTE: $\Phi_E = E(2\pi rl)$

$Q_{encl} = \alpha l$ (the charge on the length l of the line of charge)

$\Phi_E = \frac{Q_{encl}}{\epsilon_0}$ gives $E(2\pi rl) = \frac{\alpha l}{\epsilon_0}$

$E = \frac{\alpha}{2\pi\epsilon_0 r}$. The enclosed charge is positive so the direction of $\vec{E}$ is radially outward.

(ii) <u>$a < r < b$:</u> Points in this region are within the conducting tube, so $E = 0$.

(iii) **SET UP:** <u>$r > b$:</u> Apply Gauss's law to a cylindrical Gaussian surface of length l and radius r, where $r > b$, as sketched in Figure 22.39b.

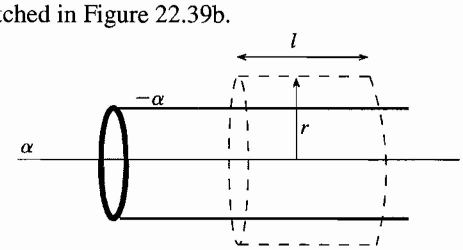


Figure 22.39b

EXECUTE: $\Phi_E = E(2\pi rl)$

$Q_{encl} = \alpha l$ (the charge on length l of the line of charge) $-\alpha l$ (the charge on length l of the tube) Thus $Q_{encl} = 0$.

$\Phi_E = \frac{Q_{encl}}{\epsilon_0}$ gives $E(2\pi rl) = 0$ and $E = 0$. The graph of E versus r is sketched in Figure 22.39c.

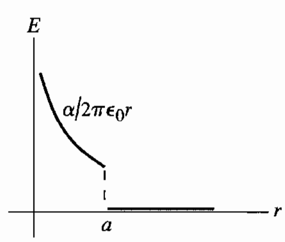


Figure 22.39c

(b) IDENTIFY: Apply Gauss's law to cylindrical surfaces that lie just outside the inner and outer surfaces of the tube. We know E so can calculate Q_{encl}.

(i) **SET UP:** <u>inner surface</u>

Apply Gauss's law to a cylindrical Gaussian surface of length l and radius r, where $a < r < b$.

EXECUTE: This surface lies within the conductor of the tube, where $E = 0$, so $\Phi_E = 0$. Then by Gauss's law $Q_{\text{encl}} = 0$. The surface encloses charge αl on the line of charge so must enclose charge $-\alpha l$ on the inner surface of the tube. The charge per unit length on the inner surface of the tube is $-\alpha$.

(ii) outer surface

The net charge per unit length on the tube is $-\alpha$. We have shown in part (i) that this must all reside on the inner surface, so there is no net charge on the outer surface of the tube.

EVALUATE: For $r < a$ the electric field is due only to the line of charge. For $r > b$ the electric field of the tube is the same as for a line of charge along its axis. The fields of the line of charge and of the tube are equal in magnitude and opposite in direction and sum to zero. For $r < a$ the electric field lines originate on the line of charge and terminate on the surface charge on the inner surface of the tube. There is no electric field outside the tube and no surface charge on the outer surface of the tube.

22.40. **IDENTIFY:** Apply Gauss's law.

SET UP: Use a Gaussian surface that is a cylinder of radius r and length l, and that is coaxial with the cylindrical charge distributions. The volume of the Gaussian cylinder is $\pi r^2 l$ and the area of its curved surface is $2\pi r l$. The charge on a length l of the charge distribution is $q = \lambda l$, where $\lambda = \rho\pi R^2$.

EXECUTE: **(a)** For $r < R$, $Q_{\text{encl}} = \rho\pi r^2 l$ and Gauss's law gives $E(2\pi r l) = \dfrac{Q_{\text{encl}}}{\epsilon_0} = \dfrac{\rho\pi r^2 l}{\epsilon_0}$ and $E = \dfrac{\rho r}{2\epsilon_0}$, radially outward.

(b) For $r > R$, $Q_{\text{encl}} = \lambda l = \rho\pi R^2 l$ and Gauss's law gives $E(2\pi r l) = \dfrac{q}{\epsilon_0} = \dfrac{\rho\pi R^2 l}{\epsilon_0}$ and $E = \dfrac{\rho R^2}{2\epsilon_0 r} = \dfrac{\lambda}{2\pi\epsilon_0 r}$, radially outward.

(c) At $r = R$, the electric field for BOTH regions is $E = \dfrac{\rho R}{2\epsilon_0}$, so they are consistent.

(d) The graph of E versus r is sketched in Figure 22.40.

EVALUATE: For $r > R$ the field is the same as for a line of charge along the axis of the cylinder.

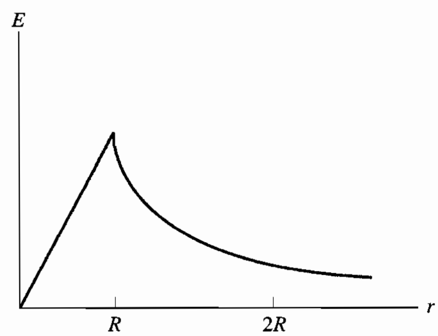


Figure 22.40

22.41. **IDENTIFY:** First make a free-body diagram of the sphere. The electric force acts to the left on it since the electric field due to the sheet is horizontal. Since it hangs at rest, the sphere is in equilibrium so the forces on it add to zero, by Newton's first law. Balance horizontal and vertical force components separately.

SET UP: Call T the tension in the thread and E the electric field. Balancing horizontal forces gives $T\sin\theta = qE$. Balancing vertical forces we get $T\cos\theta = mg$. Combining these equations gives $\tan\theta = qE/mg$, which means that $\theta = \arctan(qE/mg)$. The electric field for a sheet of charge is $E = \sigma/2\epsilon_0$.

EXECUTE: Substituting the numbers gives us $E = \dfrac{\sigma}{2\epsilon_0} = \dfrac{2.50\times10^{-9}\ \text{C/m}^2}{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)} = 1.41\times10^2\ \text{N/C}$. Then

$$\theta = \arctan\left[\frac{\left(5.00\times10^{-8}\ \text{C}\right)\left(1.41\times10^2\ \text{N/C}\right)}{\left(2.00\times10^{-6}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)}\right] = 19.8^\circ$$

EVALUATE: Increasing the field, or decreasing the mass of the sphere, would cause the sphere to hang at a larger angle.

22.42. **IDENTIFY:** Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius r and that is concentric with the conducting spheres.

EXECUTE: **(a)** For $r < a$, $E = 0$, since these points are within the conducting material.

For $a < r < b$, $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$, since there is $+q$ inside a radius r.

For $b < r < c$, $E = 0$, since since these points are within the conducting material

For $r > c$, $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$, since again the total charge enclosed is $+q$.

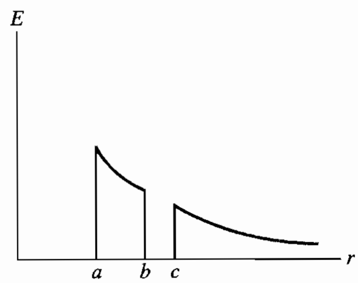
(b) The graph of E versus r is sketched in Figure 22.42a.

(c) Since the Gaussian sphere of radius r, for $b < r < c$, must enclose zero net charge, the charge on inner shell surface is $-q$.

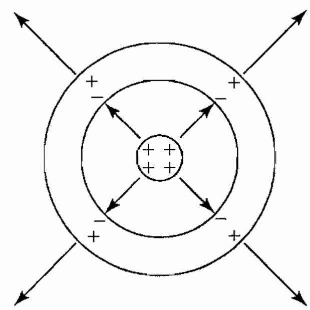
(d) Since the hollow sphere has no net charge and has charge $-q$ on its inner surface, the charge on outer shell surface is $+q$.

(e) The field lines are sketched in Figure 22.42b. Where the field is nonzero, it is radially outward.

EVALUATE: The net charge on the inner solid conducting sphere is on the surface of that sphere. The presence of the hollow sphere does not affect the electric field in the region $r < b$.



(a)



(b)

Figure 22.42

22.43. **IDENTIFY:** Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius r and that is concentric with the charge distributions.

EXECUTE: **(a)** For $r < R$, $E = 0$, since these points are within the conducting material. For $R < r < 2R$, $E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$, since the charge enclosed is Q. For $r > 2R$, $E = \frac{1}{4\pi\epsilon_0}\frac{2Q}{r^2}$ since the charge enclosed is $2Q$.

(b) The graph of E versus r is sketched in Figure 22.43.

EVALUATE: For $r < 2R$ the electric field is unaffected by the presence of the charged shell.

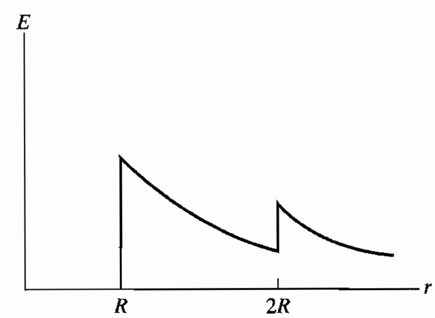


Figure 22.43

22.44. **IDENTIFY:** Apply Gauss's law and conservation of charge.

SET UP: Use a Gaussian surface that is a sphere of radius r and that has the point charge at its center.

EXECUTE: **(a)** For $r < a$, $E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$, radially outward, since the charge enclosed is Q, the charge of the point charge. For $a < r < b$, $E = 0$ since these points are within the conducting material. For $r > b$, $E = \frac{1}{4\pi\epsilon_0}\frac{2Q}{r^2}$, radially inward, since the total enclosed charge is $-2Q$.

(b) Since a Gaussian surface with radius r, for $a < r < b$, must enclose zero net charge, the total charge on the inner surface is $-Q$ and the surface charge density on inner surface is $\sigma = -\dfrac{Q}{4\pi a^2}$.

(c) Since the net charge on the shell is $-3Q$ and there is $-Q$ on the inner surface, there must be $-2Q$ on the outer surface. The surface charge density on the outer surface is $\sigma = -\dfrac{2Q}{4\pi b^2}$.

(d) The field lines and the locations of the charges are sketched in Figure 22.44a.

(e) The graph of E versus r is sketched in Figure 22.44b.

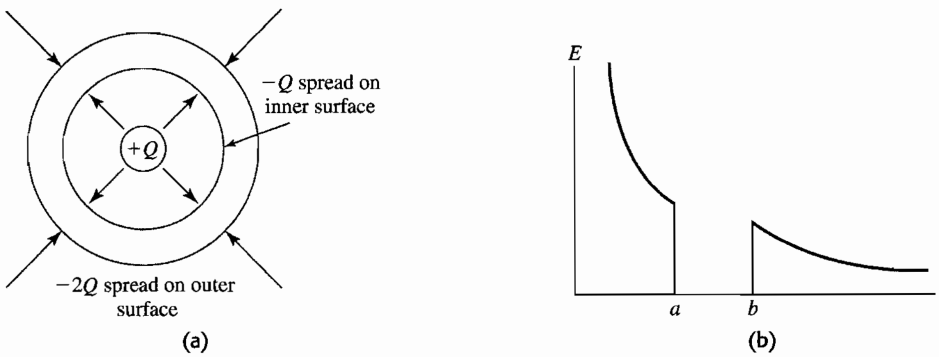
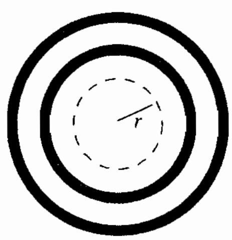


Figure 22.44

EVALUATE: For $r < a$ the electric field is due solely to the point charge Q. For $r > b$ the electric field is due to the charge $-2Q$ that is on the outer surface of the shell.

22.45. **IDENTIFY:** Apply Gauss's law to a spherical Gaussian surface with radius r. Calculate the electric field at the surface of the Gaussian sphere.

(a) SET UP: (i) $r < a$: The Gaussian surface is sketched in Figure 22.45a.



EXECUTE: $\Phi_E = EA = E(4\pi r^2)$

$Q_{\text{encl}} = 0$; no charge is enclosed

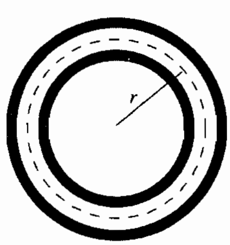
$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ says $E(4\pi r^2) = 0$ and $E = 0$.

Figure 22.45a

(ii) $a < r < b$: Points in this region are in the conductor of the small shell, so $E = 0$.

(iii) **SET UP:** $b < r < c$: The Gaussian surface is sketched in Figure 22.45b.

Apply Gauss's law to a spherical Gaussian surface with radius $b < r < c$.



EXECUTE: $\Phi_E = EA = E(4\pi r^2)$

The Gaussian surface encloses all of the small shell and none of the large shell, so $Q_{\text{encl}} = +2q$.

Figure 22.45b

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(4\pi r^2) = \dfrac{2q}{\epsilon_0}$ so $E = \dfrac{2q}{4\pi\epsilon_0 r^2}$. Since the enclosed charge is positive the electric field is radially outward.

(iv) $c < r < d$: Points in this region are in the conductor of the large shell, so $E = 0$.

(v) **SET UP:** $r > d$: Apply Gauss's law to a spherical Gaussian surface with radius $r > d$, as shown in Figure 22.45c.

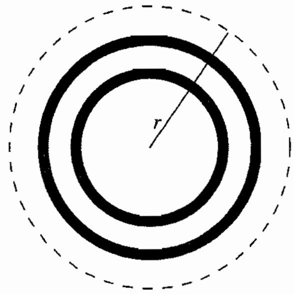


Figure 22.45c

EXECUTE: $\Phi_E = EA = E(4\pi r^2)$

The Gaussian surface encloses all of the small shell and all of the large shell, so $Q_{\text{encl}} = +2q + 4q = 6q$.

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(4\pi r^2) = \dfrac{6q}{\epsilon_0}$

$E = \dfrac{6q}{4\pi\epsilon_0 r^2}$. Since the enclosed charge is positive the electric field is radially outward.

The graph of E versus r is sketched in Figure 22.45d.

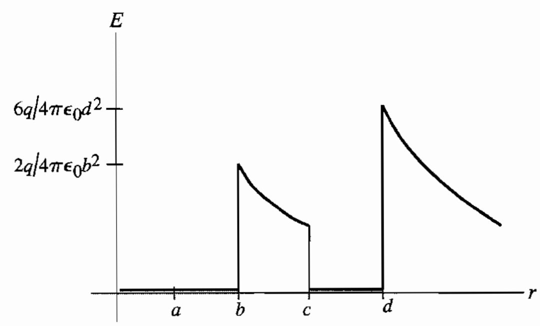


Figure 22.45d

(b) IDENTIFY and SET UP: Apply Gauss's law to a sphere that lies outside the surface of the shell for which we want to find the surface charge.

EXECUTE: (i) charge on inner surface of the small shell: Apply Gauss's law to a spherical Gaussian surface with radius $a < r < b$. This surface lies within the conductor of the small shell, where $E = 0$, so $\Phi_E = 0$. Thus by Gauss's law $Q_{\text{encl}} = 0$, so there is zero charge on the inner surface of the small shell.

(ii) charge on outer surface of the small shell: The total charge on the small shell is $+2q$. We found in part (i) that there is zero charge on the inner surface of the shell, so all $+2q$ must reside on the outer surface.

(iii) charge on inner surface of large shell: Apply Gauss's law to a spherical Gaussian surface with radius $c < r < d$. The surface lies within the conductor of the large shell, where $E = 0$, so $\Phi_E = 0$. Thus by Gauss's law $Q_{\text{encl}} = 0$. The surface encloses the $+2q$ on the small shell so there must be charge $-2q$ on the inner surface of the large shell to make the total enclosed charge zero.

(iv) charge on outer surface of large shell: The total charge on the large shell is $+4q$. We showed in part (iii) that the charge on the inner surface is $-2q$, so there must be $+6q$ on the outer surface.

EVALUATE: The electric field lines for $b < r < c$ originate from the surface charge on the outer surface of the inner shell and all terminate on the surface charge on the inner surface of the outer shell. These surface charges have equal magnitude and opposite sign. The electric field lines for $r > d$ originate from the surface charge on the outer surface of the outer sphere.

22.46. IDENTIFY: Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius r and that is concentric with the charged shells.

EXECUTE: **(a)** (i) For $r < a$, $E = 0$, since the charge enclosed is zero. (ii) For $a < r < b, E = 0$, since the points are within the conducting material. (iii) For $b < r < c, E = \dfrac{1}{4\pi\epsilon_0}\dfrac{2q}{r^2}$, outward, since charge enclosed is $+2q$.

(iv) For $c < r < d, E = 0$, since the points are within the conducting material. (v) For $r > d, E = 0$, since the net charge enclosed is zero. The graph of E versus r is sketched in Figure 22.46.

(b) (i) small shell inner surface: Since a Gaussian surface with radius *r*, for $a < r < b$, must enclose zero net charge, the charge on this surface is zero. (ii) small shell outer surface: $+2q$. (iii) large shell inner surface: Since a Gaussian surface with radius *r*, for $c < r < d$, must enclose zero net charge, the charge on this surface is $-2q$. (iv) large shell outer surface: Since there is $-2q$ on the inner surface and the total charge on this conductor is $-2q$, the charge on this surface is zero.

EVALUATE: The outer shell has no effect on the electric field for $r < c$. For $r > d$ the electric field is due only to the charge on the outer surface of the larger shell.

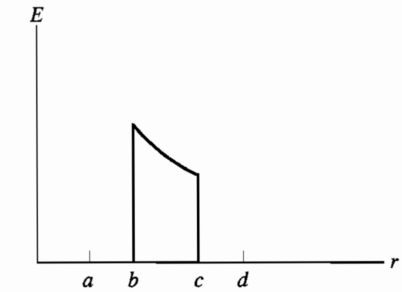


Figure 22.46

22.47. **IDENTIFY:** Apply Gauss's law

SET UP: Use a Gaussian surface that is a sphere of radius *r* and that is concentric with the charged shells.

EXECUTE: **(a)** (i) For $r < a$, $E = 0$, since charge enclosed is zero. (ii) $a < r < b$, $E = 0$, since the points are within the conducting material. (iii) For $b < r < c$, $E = \frac{1}{4\pi\epsilon_0}\frac{2q}{r^2}$, outward, since charge enclosed is $+2q$. (iv) For $c < r < d$, $E = 0$, since the points are within the conducting material. (v) For $r > d$, $E = \frac{1}{4\pi\epsilon_0}\frac{2q}{r^2}$, inward, since charge enclosed is $-2q$. The graph of the radial component of the electric field versus *r* is sketched in Figure 22.47, where we use the convention that outward field is positive and inward field is negative.

(b) (i) small shell inner surface: Since a Gaussian surface with radius *r*, for $a < r < b$, must enclose zero net charge, the charge on this surface is zero. (ii) small shell outer surface: $+2q$. (iii) large shell inner surface: Since a Gaussian surface with radius *r*, for $c < r < d$, must enclose zero net charge, the charge on this surface is $-2q$. (iv) large shell outer surface: Since there is $-2q$ on the inner surface and the total charge on this conductor is $-4q$, the charge on this surface is $-2q$.

EVALUATE: The outer shell has no effect on the electric field for $r < c$. For $r > d$ the electric field is due only to the charge on the outer surface of the larger shell.

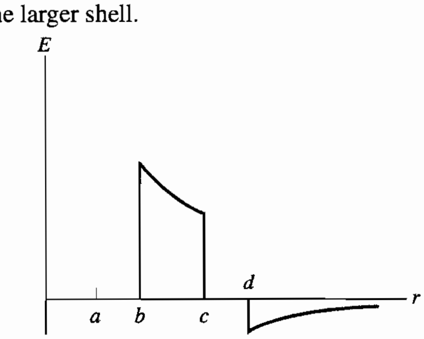


Figure 22.47

22.48. **IDENTIFY:** Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius *r* and that is concentric with the sphere and shell. The volume of the insulating shell is $V = \frac{4}{3}\pi\left([2R]^3 - R^3\right) = \frac{28\pi}{3}R^3$.

EXECUTE: **(a)** Zero net charge requires that $-Q = \frac{28\pi\rho R^3}{3}$, so $\rho = -\frac{3Q}{28\pi R^3}$.

(b) For $r < R$, $E = 0$ since this region is within the conducting sphere. For $r > 2R$, $E = 0$, since the net charge enclosed by the Gaussian surface with this radius is zero. For $R < r < 2R$, Gauss's law gives $E(4\pi r^2) = \frac{Q}{\epsilon_0} + \frac{4\pi\rho}{3\epsilon_0}(r^3 - R^3)$ and $E = \frac{Q}{4\pi\epsilon_0 r^2} + \frac{\rho}{3\epsilon_0 r^2}(r^3 - R^3)$. Substituting ρ from part (a) gives $E = \frac{2}{7\pi\epsilon_0}\frac{Q}{r^2} - \frac{Qr}{28\pi\epsilon_0 R^3}$. The net enclosed charge for each *r* in this range is positive and the electric field is outward.

(c) The graph is sketched in Figure 22.48. We see a discontinuity in going from the conducting sphere to the insulator due to the thin surface charge of the conducting sphere. But we see a smooth transition from the uniform insulator to the surrounding space.

EVALUATE: The expression for E within the insulator gives $E = 0$ at $r = 2R$.

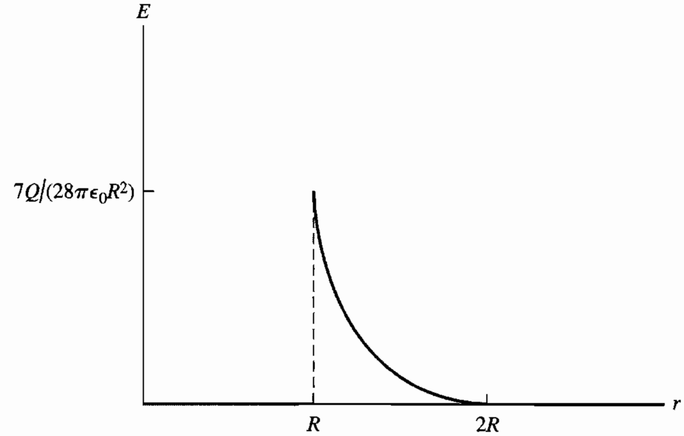


Figure 22.48

22.49. **IDENTIFY:** Use Gauss's law to find the electric field $\vec{E}$ produced by the shell for $r < R$ and $r > R$ and then use $\vec{F} = q\vec{E}$ to find the force the shell exerts on the point charge.

(a) SET UP: Apply Gauss's law to a spherical Gaussian surface that has radius $r > R$ and that is concentric with the shell, as sketched in Figure 22.49a.

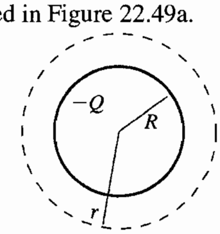


Figure 22.49a

EXECUTE: $\Phi_E = E(4\pi r^2)$

$Q_{\text{encl}} = -Q$

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(4\pi r^2) = \dfrac{-Q}{\epsilon_0}$

The magnitude of the field is $E = \dfrac{Q}{4\pi\epsilon_0 r^2}$ and it is directed toward the center of the shell. Then $F = qE = \dfrac{qQ}{4\pi\epsilon_0 r^2}$, directed toward the center of the shell. (Since q is positive, $\vec{E}$ and $\vec{F}$ are in the same direction.)

(b) SET UP: Apply Gauss's law to a spherical Gaussian surface that has radius $r < R$ and that is concentric with the shell, as sketched in Figure 22.49b.

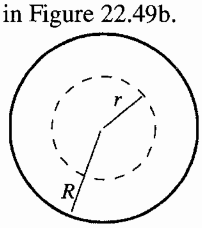


Figure 22.49b

EXECUTE: $\Phi_E = E(4\pi r^2)$

$Q_{\text{encl}} = 0$

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(4\pi r^2) = 0$

Then $E = 0$ so $F = 0$.

EVALUATE: Outside the shell the electric field and the force it exerts is the same as for a point charge $-Q$ located at the center of the shell. Inside the shell $E = 0$ and there is no force.

22.50. **IDENTIFY:** The method of Example 22.9 shows that the electric field outside the sphere is the same as for a point charge of the same charge located at the center of the sphere.

SET UP: The charge of an electron has magnitude $e = 1.60 \times 10^{-19}\,\text{C}$.

EXECUTE: (a) $E = k\dfrac{|q|}{r^2}$. For $r = R = 0.150\text{ m}$, $E = 1150\text{ N/C}$ so $|q| = \dfrac{Er^2}{k} = \dfrac{(1150\text{ N/C})(0.150\text{ m})^2}{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2} = 2.88\times10^{-9}\text{ C}$.

The number of excess electrons is $\dfrac{2.88\times10^{-9}\text{C}}{1.60\times10^{-19}\text{C/electron}} = 1.80\times10^{10}$ electrons.

(b) $r = R + 0.100\text{ m} = 0.250\text{ m}$. $E = k\dfrac{|q|}{r^2} = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\dfrac{2.88\times10^{-9}\text{ C}}{(0.250\text{ m})^2} = 414\text{ N/C}$.

EVALUATE: The magnitude of the electric field decreases according to the square of the distance from the center of the sphere.

22.51. **IDENTIFY:** The net electric field is the vector sum of the fields due to the sheet of charge on each surface of the plate.

SET UP: The electric field due to the sheet of charge on each surface is $E = \sigma/2\epsilon_0$ and is directed away from the surface.

EXECUTE: **(a)** For the conductor the charge sheet on each surface produces fields of magnitude $\sigma/2\epsilon_0$ and in the same direction, so the total field is twice this, or σ/ϵ_0.

(b) At points inside the plate the fields of the sheets of charge on each surface are equal in magnitude and opposite in direction, so their vector sum is zero. At points outside the plate, on either side, the fields of the two sheets of charge are in the same direction so their magnitudes add, giving $E = \sigma/\epsilon_0$.

EVALUATE: Gauss's law can also be used directly to determine the fields in these regions.

22.52. **IDENTIFY:** Example 22.9 gives the expression for the electric field both inside and outside a uniformly charged sphere. Use $\vec{F} = -e\vec{E}$ to calculate the force on the electron.

SET UP: The sphere has charge $Q = +e$.

EXECUTE: **(a)** Only at $r = 0$ is $E = 0$ for the uniformly charged sphere.

(b) At points inside the sphere, $E_r = \dfrac{er}{4\pi\epsilon_0 R^3}$. The field is radially outward. $F_r = -eE = -\dfrac{1}{4\pi\epsilon_0}\dfrac{e^2 r}{R^3}$. The minus sign denotes that F_r is radially inward. For simple harmonic motion, $F_r = -kr = -m\omega^2 r$, where $\omega = \sqrt{k/m} = 2\pi f$.

$F_r = -m\omega^2 r = -\dfrac{1}{4\pi\epsilon_0}\dfrac{e^2 r}{R^3}$ so $\omega = \sqrt{\dfrac{1}{4\pi\epsilon_0}\dfrac{e^2}{mR^3}}$ and $f = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{4\pi\epsilon_0}\dfrac{e^2}{mR^3}}$.

(c) If $f = 4.57\times10^{14}\text{ Hz} = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{4\pi\epsilon_0}\dfrac{e^2}{mR^3}}$ then $R = \sqrt[3]{\dfrac{1}{4\pi\epsilon_0}\dfrac{(1.60\times10^{-19}\text{ C})^2}{4\pi^2(9.11\times10^{-31}\text{ kg})(4.57\times10^{14}\text{ Hz})^2}} = 3.13\times10^{-10}\text{ m}$.

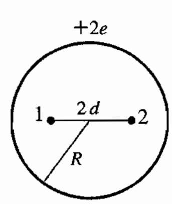
The atom radius in this model is the correct order of magnitude.

(d) If $r > R$, $E_r = \dfrac{e}{4\pi\epsilon_0 r^2}$ and $F_r = -\dfrac{e^2}{4\pi\epsilon_0 r^2}$. The electron would still oscillate because the force is directed toward the equilibrium position at $r = 0$. But the motion would not be simple harmonic, since F_r is proportional to $1/r^2$ and simple harmonic motion requires that the restoring force be proportional to the displacement from equilibrium.

EVALUATE: As long as the initial displacement is less than R the frequency of the motion is independent of the initial displacement.

22.53. **IDENTIFY:** There is a force on each electron due to the other electron and a force due to the sphere of charge. Use Coulomb's law for the force between the electrons. Example 22.9 gives E inside a uniform sphere and Eq.(21.3) gives the force.

SET UP: The positions of the electrons are sketched in Figure 22.53a.



If the electrons are in equilibrium the net force on each one is zero.

Figure 22.53a

EXECUTE: Consider the forces on electron 2. There is a repulsive force F_1 due to the other electron, electron 1.

$$F_1 = \frac{1}{4\pi\epsilon_0}\frac{e^2}{(2d)^2}$$

The electric field inside the uniform distribution of positive charge is $E = \dfrac{Qr}{4\pi\epsilon_0 R^3}$ (Example 22.9), where $Q = +2e$.

At the position of electron 2, $r = d$. The force F_{cd} exerted by the positive charge distribution is $F_{cd} = eE = \dfrac{e(2e)d}{4\pi\epsilon_0 R^3}$ and is attractive.

The force diagram for electron 2 is given in Figure 22.53b.

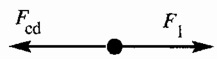


Figure 22.53b

Net force equals zero implies $F_1 = F_{cd}$ and $\dfrac{1}{4\pi\epsilon_0}\dfrac{e^2}{4d^2} = \dfrac{2e^2 d}{4\pi\epsilon_0 R^3}$

Thus $(1/4d^2) = 2d/R^3$, so $d^3 = R^3/8$ and $d = R/2$.

EVALUATE: The electric field of the sphere is radially outward; it is zero at the center of the sphere and increases with distance from the center. The force this field exerts on one of the electrons is radially inward and increases as the electron is farther from the center. The force from the other electron is radially outward, is infinite when $d = 0$ and decreases as d increases. It is reasonable therefore for there to be a value of d for which these forces balance.

22.54. **IDENTIFY:** Use Gauss's law to find the electric field both inside and outside the slab.

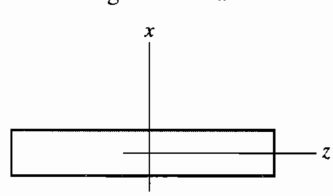
SET UP: Use a Gaussian surface that has one face of area A in the yz plane at $x = 0$, and the other face at a general value x. The volume enclosed by such a Gaussian surface is Ax.

EXECUTE: **(a)** The electric field of the slab must be zero by symmetry. There is no preferred direction in the yz plane, so the electric field can only point in the x-direction. But at the origin, neither the positive nor negative x-directions should be singled out as special, and so the field must be zero.

(b) For $|x| \le d$, Gauss's law gives $EA = \dfrac{Q_{encl}}{\epsilon_0} = \dfrac{\rho A|x|}{\epsilon_0}$ and $E = \dfrac{\rho|x|}{\epsilon_0}$, with direction given by $\dfrac{x}{|x|}\hat{\boldsymbol{i}}$ (away from the center of the slab). Note that this expression does give $E = 0$ at $x = 0$. Outside the slab, the enclosed charge does not depend on x and is equal to ρAd. For $|x| \ge d$, Gauss's law gives $EA = \dfrac{Q_{encl}}{\epsilon_0} = \dfrac{\rho Ad}{\epsilon_0}$ and $E = \dfrac{\rho d}{\epsilon_0}$, again with direction given by $\dfrac{x}{|x|}\hat{\boldsymbol{i}}$.

EVALUATE: At the surfaces of the slab, $x = \pm d$. For these values of x the two expressions for E (for inside and outside the slab) give the same result. The charge per unit area σ of the slab is given by $\sigma A = \rho A(2d)$ and $\rho d = \sigma/2$. The result for E outside the slab can therefore be written as $E = \sigma/2\epsilon_0$ and is the same as for a thin sheet of charge.

22.55. **(a) IDENTIFY** and **SET UP:** Consider the direction of the field for x slightly greater than and slightly less than zero. The slab is sketched in Figure 22.55a.



$\rho(x) = \rho_0(x/d)^2$

Figure 22.55a

EXECUTE: The charge distribution is symmetric about $x = 0$, so by symmetry $E(x) = E(-x)$. But for $x > 0$ the field is in the $+x$ direction and for $x < 0$ the field is in the $-x$ direction. At $x = 0$ the field can't be both in the $+x$ and $-x$ directions so must be zero. That is, $E_x(x) = -E_x(-x)$. At point $x = 0$ this gives $E_x(0) = -E_x(0)$ and this equation is satisfied only for $E_x(0) = 0$.

(b) IDENTIFY and SET UP: $|x| > d$ (outside the slab)

Apply Gauss's law to a cylindrical Gaussian surface whose axis is perpendicular to the slab and whose end caps have area A and are the same distance $|x| > d$ from $x = 0$, as shown in Figure 22.55b.

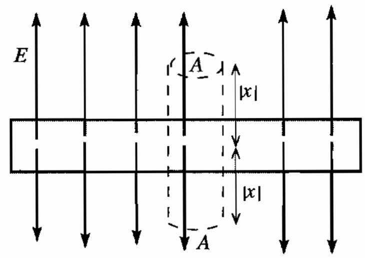


Figure 22.55b

EXECUTE: $\Phi_E = 2EA$

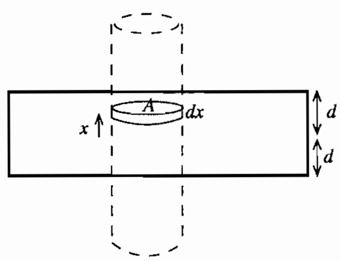


Figure 22.55c

To find Q_{encl} consider a thin disk at coordinate x and with thickness dx, as shown in Figure 22.55c. The charge within this disk is $dq = \rho dV = \rho A dx = (\rho_0 A/d^2)x^2 dx.$

The total charge enclosed by the Gaussian cylinder is

$$Q_{\text{encl}} = 2\int_0^d dq = (2\rho_0 A/d^2)\int_0^d x^2 dx = (2\rho_0 A/d^2)(d^3/3) = \tfrac{2}{3}\rho_0 A d.$$

Then $\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $2EA = 2\rho_0 A d/3\epsilon_0$.

$$E = \rho_0 d/3\epsilon_0$$

$\vec{E}$ is directed away from $x = 0$, so $\vec{E} = (\rho_0 d/3\epsilon_0)(x/|x|)\hat{\imath}$.

IDENTIFY and SET UP: $|x| < d$ (inside the slab)

Apply Gauss's law to a cylindrical Gaussian surface whose axis is perpendicular to the slab and whose end caps have area A and are the same distance $|x| < d$ from $x = 0$, as shown in Figure 22.55d.

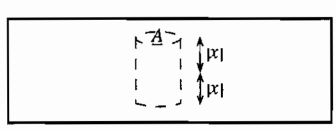


Figure 22.55d

EXECUTE: $\Phi_E = 2EA$

Q_{encl} is found as above, but now the integral on dx is only from 0 to x instead of 0 to d.

$$Q_{\text{encl}} = 2\int_0^x dq = (2\rho_0 A/d^2)\int_0^x x^2 dx = (2\rho_0 A/d^2)(x^3/3).$$

Then $\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $2EA = 2\rho_0 A x^3/3\epsilon_0 d^2$.

$$E = \rho_0 x^3/3\epsilon_0 d^2$$

$\vec{E}$ is directed away from $x = 0$, so $\vec{E} = (\rho_0 x^3/3\epsilon_0 d^2)\hat{\imath}$.

EVALUATE: Note that $E = 0$ at $x = 0$ as stated in part (a). Note also that the expressions for $|x| > d$ and $|x| < d$ agree for $x = d$.

22.56. **IDENTIFY:** Apply $\vec{F} = q\vec{E}$ to relate the force on q to the electric field at the location of q.

SET UP: Flux is negative if the electric field is directed into the enclosed volume.

EXECUTE: **(a)** We could place two charges $+Q$ on either side of the charge $+q$, as shown in Figure 22.56.

(b) In order for the charge to be stable, the electric field in a neighborhood around it must always point back to the equilibrium position.

(c) If q is moved to infinity and we require there to be an electric field always pointing in to the region where q had been, we could draw a small Gaussian surface there. We would find that we need a negative flux into the surface. That is, there has to be a negative charge in that region. However, there is none, and so we cannot get such a stable equilibrium.

(d) For a negative charge to be in stable equilibrium, we need the electric field to always point away from the charge position. The argument in (c) carries through again, this time implying that a positive charge must be in the space where the negative charge was if stable equilibrium is to be attained.

EVALUATE: If q is displaced to the left or right in Figure 22.56, the net force is directed back toward the equilibrium position. But if q is displaced slightly in a direction perpendicular to the line connecting the two charges Q, then the net force on q is directed away from the equilibrium position and the equilibrium is not stable for such a displacement.

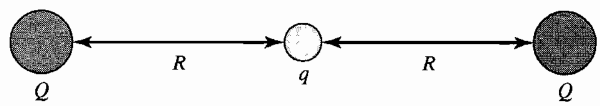


Figure 22.56

22.57. $\rho(r) = \rho_0(1 - r/R)$ for $r \le R$ where $\rho_0 = 3Q/\pi R^3$. $\rho(r) = 0$ for $r \ge R$

(a) IDENTIFY: The charge density varies with r inside the spherical volume. Divide the volume up into thin concentric shells, of radius r and thickness dr. Find the charge dq in each shell and integrate to find the total charge.

SET UP: The thin shell is sketched in Figure 22.57a.

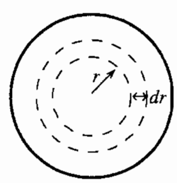


Figure 22.57a

EXECUTE: The volume of such a shell is $dV = 4\pi r^2 dr$

The charge contained within the shell is

$dq = \rho(r)dV = 4\pi r^2 \rho_0(1 - r/R)dr$

The total charge Q in the charge distribution is obtained by integrating dq over all such shells into which the sphere can be subdivided:

$$Q = \int dq = \int_0^R 4\pi r^2 \rho_0(1 - r/R)dr = 4\pi\rho_0 \int_0^R \left(r^2 - r^3/R\right)dr$$

$$Q = 4\pi\rho_0\left[\frac{r^3}{3} - \frac{r^4}{4R}\right]_0^R = 4\pi\rho_0\left(\frac{R^3}{3} - \frac{R^4}{4R}\right) = 4\pi\rho_0\left(R^3/12\right) = 4\pi\left(3Q/\pi R^3\right)\left(R^3/12\right) = Q,$$ as was to be shown.

(b) IDENTIFY: Apply Gauss's law to a spherical surface of radius r, where $r > R$.

SET UP: The Gaussian surface is shown in Figure 22.57b.

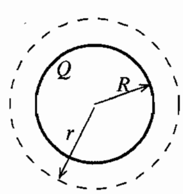


Figure 22.57b

EXECUTE: $\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$

$E\left(4\pi r^2\right) = \dfrac{Q}{\epsilon_0}$

$E = \dfrac{Q}{4\pi\epsilon_0 r^2}$; same as for point charge of charge Q.

(c) IDENTIFY: Apply Gauss's law to a spherical surface of radius r, where $r < R$:

SET UP: The Gaussian surface is shown in Figure 22.57c.

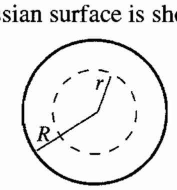


Figure 22.57c

EXECUTE: $\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$

$\Phi_E = E\left(4\pi r^2\right)$

To calculate the enclosed charge Q_{encl} use the same technique as in part (a), except integrate dq out to r rather than R. (We want the charge that is inside radius r.)

$$Q_{\text{encl}} = \int_0^r 4\pi r'^2 \rho_0 \left(1 - \frac{r'}{R}\right) dr' = 4\pi\rho_0 \int_0^r \left(r'^2 - \frac{r'^3}{R}\right) dr'$$

$$Q_{\text{encl}} = 4\pi\rho_0 \left[\frac{r'^3}{3} - \frac{r'^4}{4R}\right]_0^r = 4\pi\rho_0 \left(\frac{r^3}{3} - \frac{r^4}{4R}\right) = 4\pi\rho_0 r^3 \left(\frac{1}{3} - \frac{r}{4R}\right)$$

$$\rho_0 = \frac{3Q}{\pi R^3} \text{ so } Q_{\text{encl}} = 12Q\frac{r^3}{R^3}\left(\frac{1}{3} - \frac{r}{4R}\right) = Q\left(\frac{r^3}{R^3}\right)\left(4 - 3\frac{r}{R}\right).$$

Thus Gauss's law gives $E(4\pi r^2) = \frac{Q}{\epsilon_0}\left(\frac{r^3}{R^3}\right)\left(4 - 3\frac{r}{R}\right)$

$$E = \frac{Qr}{4\pi\epsilon_0 R^3}\left(4 - \frac{3r}{R}\right),\ r \le R$$

(**d**) The graph of E versus r is sketched in Figure 22.57d.

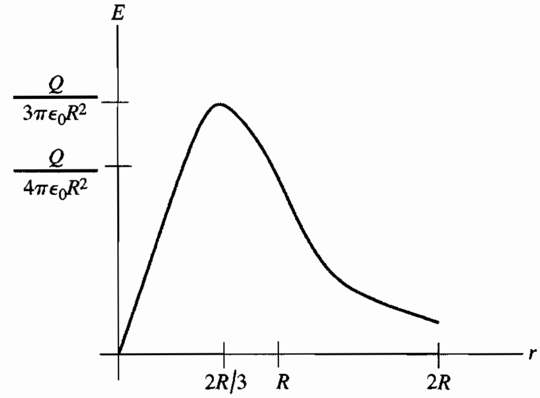


Figure 22.57d

(**e**) Where the electric field is a maximum, $\frac{dE}{dr} = 0$. Thus $\frac{d}{dr}\left(4r - \frac{3r^2}{R}\right) = 0$ so $4 - 6r/R = 0$ and $r = 2R/3$.

At this value of r, $E = \frac{Q}{4\pi\epsilon_0 R^3}\left(\frac{2R}{3}\right)\left(4 - \frac{3}{R}\frac{2R}{3}\right) = \frac{Q}{3\pi\epsilon_0 R^2}$

Evaluate: Our expressions for $E(r)$ for $r < R$ and for $r > R$ agree at $r = R$. The results of part (e) for the value of r where $E(r)$ is a maximum agrees with the graph in part (d).

22.58. Identify: Apply Gauss's law.

Set Up: Use a Gaussian surface that is a sphere of radius r and that is concentric with the spherical distribution of charge. The volume of a thin spherical shell of radius r and thickness dr is $dV = 4\pi r^2 dr$.

Execute: (**a**) $Q = \int \rho(r)dV = 4\pi \int_0^\infty \rho(r) r^2 dr = 4\pi\rho_0 \int_0^R \left(1 - \frac{4r}{3R}\right) r^2 dr = 4\pi\rho_0 \left[\int_0^R r^2 dr - \frac{4}{3R}\int_0^R r^3 dr\right]$

$Q = 4\pi\rho_0 \left[\frac{R^3}{3} - \frac{4}{3R}\frac{R^4}{4}\right] = 0$. The total charge is zero.

(**b**) For $r \ge R$, $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0} = 0$, so $E = 0$.

(**c**) For $r \le R$, $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0} = \frac{4\pi}{\epsilon_0}\int_0^r \rho(r')r'^2 dr'$. $E4\pi r^2 = \frac{4\pi\rho_0}{\epsilon_0}\left[\int_0^r r'^2 dr' - \frac{4}{3R}\int_0^r r'^3 dr'\right]$ and

$$E = \frac{\rho_0}{\epsilon_0}\frac{1}{r^2}\left[\frac{r^3}{3} - \frac{r^4}{3R}\right] = \frac{\rho_0}{3\epsilon_0} r\left[1 - \frac{r}{R}\right].$$

(**d**) The graph of E versus r is sketched in Figure 22.58.

(**e**) Where E is a maximum, $\frac{dE}{dr} = 0$. This gives $\frac{\rho_0}{3\epsilon_0} - \frac{2\rho_0 r_{\text{max}}}{3\epsilon_0 R} = 0$ and $r_{\text{max}} = \frac{R}{2}$. At this r, $E = \frac{\rho_0}{3\epsilon_0}\frac{R}{2}\left[1 - \frac{1}{2}\right] = \frac{\rho_0 R}{12\epsilon_0}$.

EVALUATE: The result in part (b) for $r \le R$ gives $E = 0$ at $r = R$; the field is continuous at the surface of the charge distribution.

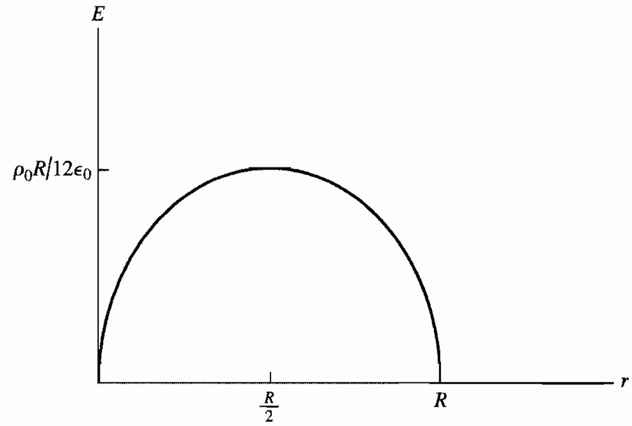


Figure 22.58

22.59. IDENTIFY: Follow the steps specified in the problem.

SET UP: In spherical polar coordinates $d\vec{A} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{r}$. $\oint \sin\theta \, d\theta \, d\phi = 4\pi$.

EXECUTE: (a) $\Phi_g = \oint \vec{g} \cdot d\vec{A} = -Gm \oint \dfrac{r^2 \sin\theta \, d\theta \, d\phi}{r^2} = -4\pi Gm.$

(b) For any closed surface, mass OUTSIDE the surface contributes zero to the flux passing through the surface. Thus the formula above holds for any situation where m is the mass enclosed by the Gaussian surface.

That is, $\Phi_g = \oint \vec{g} \cdot d\vec{A} = -4\pi G M_{\text{encl}}$.

EVALUATE: The minus sign in the expression for the flux signifies that the flux is directed inward.

22.60. IDENTIFY: Apply $\oint \vec{g} \cdot d\vec{A} = -4\pi G M_{\text{encl}}$.

SET UP: Use a Gaussian surface that is a sphere of radius r, concentric with the mass distribution. Let Φ_g denote $\oint \vec{g} \cdot d\vec{A}$

EXECUTE: (a) Use a Gaussian sphere with radius $r > R$, where R is the radius of the mass distribution. g is constant on this surface and the flux is inward. The enclosed mass is M. Therefore, $\Phi_g = -g 4\pi r^2 = -4\pi GM$ and $g = \dfrac{GM}{r^2}$, which is the same as for a point mass.

(b) For a Gaussian sphere of radius $r < R$, where R is the radius of the shell, $M_{\text{encl}} = 0$, so $g = 0$.

(c) Use a Gaussian sphere of radius $r < R$, where R is the radius of the planet. Then $M_{\text{encl}} = \rho\left(\dfrac{4}{3}\pi r^3\right) = Mr^3/R^3$.

This gives $\Phi_g = -g 4\pi r^2 = -4\pi G M_{\text{encl}} = -4\pi G\left(M \dfrac{r^3}{R^3}\right)$ and $g = \dfrac{GMr}{R^3}$, which is linear in r.

EVALUATE: The spherically symmetric distribution of mass results in an acceleration due to gravity $\vec{g}$ that is radical and that depends only on r, the distance from the center of the mass distribution.

22.61. (a) **IDENTIFY:** Use $\vec{E}(\vec{r})$ from Example (22.9) (inside the sphere) and relate the position vector of a point inside the sphere measured from the origin to that measured from the center of the sphere.

SET UP: For an insulating sphere of uniform charge density ρ and centered at the origin, the electric field inside the sphere is given by $E = Qr'/4\pi\epsilon_0 R^3$ (Example 22.9), where $\vec{r}'$ is the vector from the center of the sphere to the point where E is calculated.

But $\rho = 3Q/4\pi R^3$ so this may be written as $E = \rho r/3\epsilon_0$. And $\vec{E}$ is radially outward, in the direction of $\vec{r}'$, so $\vec{E} = \rho\vec{r}'/3\epsilon_0$.

For a sphere whose center is located by vector $\vec{b}$, a point inside the sphere and located by $\vec{r}$ is located by the vector $\vec{r}' = \vec{r} - \vec{b}$ relative to the center of the sphere, as shown in Figure 22.61.

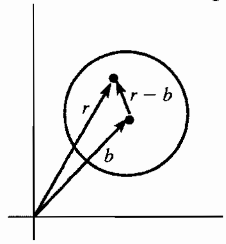


Figure 22.61

EXECUTE: Thus $\vec{E} = \dfrac{\rho(\vec{r} - \vec{b})}{3\epsilon_0}$

EVALUATE: When $b = 0$ this reduces to the result of Example 22.9. When $\vec{r} = \vec{b}$, this gives $E = 0$, which is correct since we know that $E = 0$ at the center of the sphere.

(b) IDENTIFY: The charge distribution can be represented as a uniform sphere with charge density ρ and centered at the origin added to a uniform sphere with charge density $-\rho$ and centered at $\vec{r} = \vec{b}$.

SET UP: $\vec{E} = \vec{E}_{\text{uniform}} + \vec{E}_{\text{hole}}$, where $\vec{E}_{\text{uniform}}$ is the field of a uniformly charged sphere with charge density ρ and $\vec{E}_{\text{hole}}$ is the field of a sphere located at the hole and with charge density $-\rho$. (Within the spherical hole the net charge density is $+\rho - \rho = 0$.)

EXECUTE: $\vec{E}_{\text{uniform}} = \dfrac{\rho\vec{r}}{3\epsilon_0}$, where $\vec{r}$ is a vector from the center of the sphere.

$\vec{E}_{\text{hole}} = \dfrac{-\rho(\vec{r} - \vec{b})}{3\epsilon_0}$, at points inside the hole.

Then $\vec{E} = \dfrac{\rho\vec{r}}{3\epsilon_0} + \left(\dfrac{-\rho(\vec{r} - \vec{b})}{3\epsilon_0}\right) = \dfrac{\rho\vec{b}}{3\epsilon_0}$.

EVALUATE: $\vec{E}$ is independent of $\vec{r}$ so is uniform inside the hole. The direction of $\vec{E}$ inside the hole is in the direction of the vector $\vec{b}$, the direction from the center of the insulating sphere to the center of the hole.

22.62. **IDENTIFY:** We first find the field of a cylinder off-axis, then the electric field in a hole in a cylinder is the difference between two electric fields: that of a solid cylinder on-axis, and one off-axis, at the location of the hole.

SET UP: Let $\vec{r}$ locate a point within the hole, relative to the axis of the cylinder and let $\vec{r}'$ locate this point relative to the axis of the hole. Let $\vec{b}$ locate the axis of the hole relative to the axis of the cylinder. As shown in Figure 22.62, $\vec{r}' = \vec{r} - \vec{b}$. Problem 23.48 shows that at points within a long insulating cylinder, $\vec{E} = \dfrac{\rho\vec{r}}{2\epsilon_0}$.

EXECUTE: $\vec{E}_{\text{off-axis}} = \dfrac{\rho\vec{r}'}{2\epsilon_0} = \dfrac{\rho(\vec{r} - \vec{b})}{2\epsilon_0}$. $\vec{E}_{\text{hole}} = \vec{E}_{\text{cylinder}} - \vec{E}_{\text{off-axis}} = \dfrac{\rho\,\vec{r}}{2\epsilon_0} - \dfrac{\rho(\vec{r} - \vec{b})}{2\epsilon_0} = \dfrac{\rho\,\vec{b}}{2\epsilon_0}$.

Note that $\vec{E}$ is uniform.

EVALUATE: If the hole is coaxial with the cylinder, $b = 0$ and $E_{\text{hole}} = 0$.

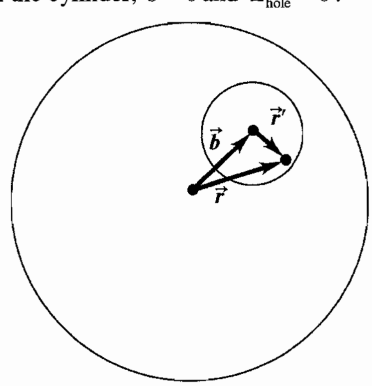


Figure 22.62

22.63. **IDENTIFY:** The electric field at each point is the vector sum of the fields of the two charge distributions.

SET UP: Inside a sphere of uniform positive charge, $E = \dfrac{\rho r}{3\epsilon_0}$.

$\rho = \dfrac{Q}{\frac{4}{3}\pi R^3} = \dfrac{3Q}{4\pi R^3}$ so $E = \dfrac{Qr}{4\pi\epsilon_0 R^3}$, directed away from the center of the sphere. Outside a sphere of uniform positive charge, $E = \dfrac{Q}{4\pi\epsilon_0 r^2}$, directed away from the center of the sphere.

EXECUTE: **(a)** $x = 0$. This point is inside sphere 1 and outside sphere 2. The fields are shown in Figure 22.63a.

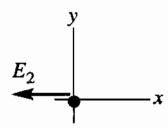


Figure 22.63a

$E_1 = \dfrac{Qr}{4\pi\epsilon_0 R^3} = 0$, since $r = 0$.

$E_2 = \dfrac{Q}{4\pi\epsilon_0 r^2}$ with $r = 2R$ so $E_2 = \dfrac{Q}{16\pi\epsilon_0 R^2}$, in the $-x$-direction.

Thus $\vec{E} = \vec{E}_1 + \vec{E}_2 = -\dfrac{Q}{16\pi\epsilon_0 R^2}\hat{i}$.

(b) $x = R/2$. This point is inside sphere 1 and outside sphere 2. Each field is directed away from the center of the sphere that produces it. The fields are shown in Figure 22.63b.

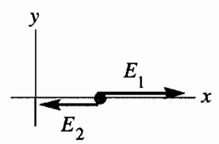


Figure 22.63b

$E_1 = \dfrac{Qr}{4\pi\epsilon_0 R^3}$ with $r = R/2$ so

$E_1 = \dfrac{Q}{8\pi\epsilon_0 R^2}$

$E_2 = \dfrac{Q}{4\pi\epsilon_0 r^2}$ with $r = 3R/2$ so $E_2 = \dfrac{Q}{9\pi\epsilon_0 R^2}$

$E = E_1 - E_2 = \dfrac{Q}{72\pi\epsilon_0 R^2}$, in the $+x$-direction and $\vec{E} = \dfrac{Q}{72\pi\epsilon_0 R^2}\hat{i}$

(c) $x = R$. This point is at the surface of each sphere. The fields have equal magnitudes and opposite directions, so $E = 0$.

(d) $x = 3R$. This point is outside both spheres. Each field is directed away from the center of the sphere that produces it. The fields are shown in Figure 22.63c.

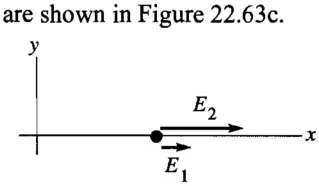


Figure 22.63c

$E_1 = \dfrac{Q}{4\pi\epsilon_0 r^2}$ with $r = 3R$ so

$E_1 = \dfrac{Q}{36\pi\epsilon_0 R^2}$

$E_2 = \dfrac{Q}{4\pi\epsilon_0 r^2}$ with $r = R$ so $E_2 = \dfrac{Q}{4\pi\epsilon_0 R^2}$

$E = E_1 + E_2 = \dfrac{5Q}{18\pi\epsilon_0 R^2}$, in the $+x$-direction and $\vec{E} = \dfrac{5Q}{18\pi\epsilon_0 R^2}\hat{i}$

EVALUATE: The field of each sphere is radially outward from the center of the sphere. We must use the correct expression for $E(r)$ for each sphere, depending on whether the field point is inside or outside that sphere.

22.64. **IDENTIFY:** The net electric field at any point is the vector sum of the fields at each sphere.

SET UP: Example 22.9 gives the electric field inside and outside a uniformly charged sphere. For the positively charged sphere the field is radially outward and for the negatively charged sphere the electric field is radially inward.

EXECUTE: (**a**) At this point the field of the left-hand sphere is zero and the field of the right-hand sphere is toward the center of that sphere, in the +*x*-direction. This point is outside the right-hand sphere, a distance $r = 2R$ from its center. $\vec{E} = +\dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{4R^2}\hat{i}$.

(**b**) This point is inside the left-hand sphere, at $r = R/2$, and is outside the right-hand sphere, at $r = 3R/2$. Both fields are in the +*x*-direction.

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\left[\frac{Q(R/2)}{R^3} + \frac{Q}{(3R/2)^2}\right]\hat{i} = \frac{1}{4\pi\epsilon_0}\left[\frac{Q}{2R^2} + \frac{4Q}{9R^2}\right]\hat{i} = \frac{1}{4\pi\epsilon_0}\frac{17Q}{18R^2}\hat{i}.$$

(**c**) This point is outside both spheres, at a distance $r = R$ from their centers. Both fields are in the +*x*-direction. $\vec{E} = \dfrac{1}{4\pi\epsilon_0}\left[\dfrac{Q}{R^2} + \dfrac{Q}{R^2}\right]\hat{i} = \dfrac{Q}{2\pi\epsilon_0 R^2}\hat{i}$.

(**d**) This point is outside both spheres, a distance $r = 3R$ from the center of the left-hand sphere and a distance $r = R$ from the center of the right-hand sphere. The field of the left-hand sphere is in the +*x*-direction and the field of the right-hand sphere is in the $-x$-direction. $\vec{E} = \dfrac{1}{4\pi\epsilon_0}\left[\dfrac{Q}{(3R)^2} - \dfrac{Q}{R^2}\right]\hat{i} = \dfrac{1}{4\pi\epsilon_0}\left[\dfrac{Q}{9R^2} - \dfrac{Q}{R^2}\right]\hat{i} = \dfrac{-1}{4\pi\epsilon_0}\dfrac{8Q}{9R^2}\hat{i}$.

EVALUATE: At all points on the *x*-axis the net field is parallel to the *x*-axis.

22.65. IDENTIFY: Let $-dQ$ be the electron charge contained within a spherical shell of radius r' and thickness dr'. Integrate r' from 0 to *r* to find the electron charge within a sphere of radius *r*. Apply Gauss's law to a sphere of radius *r* to find the electric field $E(r)$.

SET UP: The volume of the spherical shell is $dV = 4\pi r'^2\, dr'$.

EXECUTE: (**a**) $Q(r) = Q - \int \rho dV = Q - \dfrac{Q4\pi}{\pi a_0^3}\int e^{-2r/a_0} r^2 dr = Q - \dfrac{4Q}{a_0^3}\int_0^r r'^2 e^{-2r'/a_0}\, dr'$.

$$Q(r) = Q - \frac{4Qe^{-\alpha r}}{a_0^3\alpha^3}(2e^{\alpha r} - \alpha^2 r^2 - 2\alpha r - 2) = Qe^{-2r/a_0}[2(r/a_0)^2 + 2(r/a_0) + 1].$$

Note if $r \to \infty$, $Q(r) \to 0$; the total net charge of the atom is zero.

(**b**) The electric field is radially outward. Gauss's law gives $E(4\pi r^2) = \dfrac{Q(r)}{\epsilon_0}$ and $E = \dfrac{kQe^{-2r/a_0}}{r^2}(2(r/a_0)^2 + 2(r/a_0) + 1)$.

(**c**) The graph of *E* versus *r* is sketched in Figure 22.65. What is plotted is the scaled *E*, equal to $E/E_{\text{pt charge}}$, versus scaled *r*, equal to r/a_0. $E_{\text{pt charge}} = \dfrac{kQ}{r^2}$ is the field of a point charge.

EVALUATE: As $r \to 0$, the field approaches that of the positive point charge (the proton). For increasing *r* the electric field falls to zero more rapidly than the $1/r^2$ dependence for a point charge.

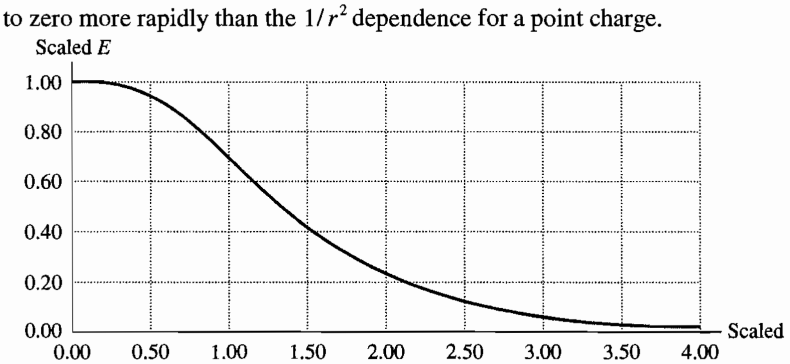


Figure 22.65

22.66. IDENTIFY: The charge in a spherical shell of radius *r* and thickness *dr* is $dQ = \rho(r)4\pi r^2\, dr$. Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius *r*. Let Q_i be the charge in the region $r \le R/2$ and let Q_0 be the charge in the region where $R/2 \le r \le R$.

EXECUTE: (a) The total charge is $Q = Q_i + Q_0$, where $Q_i = \alpha\dfrac{4\pi(R/2)^3}{3} = \dfrac{\alpha\pi R^3}{6}$ and

$$Q_0 = 4\pi(2\alpha)\int_{R/2}^{R}(r^2 - r^3/R)dr = 8\alpha\pi\left(\frac{(R^3 - R^3/8)}{3} - \frac{(R^4 - R^4/16)}{4R}\right) = \frac{11\alpha\pi R^3}{24}.$$ Therefore, $Q = \dfrac{15\alpha\pi R^3}{24}$ and $\alpha = \dfrac{8Q}{5\pi R^3}$.

(b) For $r \le R/2$, Gauss's law gives $E4\pi r^2 = \dfrac{\alpha 4\pi r^3}{3\epsilon_0}$ and $E = \dfrac{\alpha r}{3\epsilon_0} = \dfrac{8Qr}{15\pi\epsilon_0 R^3}$. For $R/2 \le r \le R$,

$$E4\pi r^2 = \frac{Q_i}{\epsilon_0} + \frac{1}{\epsilon_0}\left(8\alpha\pi\left(\frac{(r^3 - R^3/8)}{3} - \frac{(r^4 - R^4/16)}{4R}\right)\right)$$ and

$$E = \frac{\alpha\pi R^3}{24\epsilon_0(4\pi r^2)}(64(r/R)^3 - 48(r/R)^4 - 1) = \frac{kQ}{15r^2}(64(r/R)^3 - 48(r/R)^4 - 1).$$

For $r \ge R$, $E(4\pi r^2) = \dfrac{Q}{\epsilon_0}$ and $E = \dfrac{Q}{4\pi\epsilon_0 r^2}$.

(c) $\dfrac{Q_i}{Q} = \dfrac{(4Q/15)}{Q} = \dfrac{4}{15} = 0.267.$

(d) For $r \le R/2$, $F_r = -eE = -\dfrac{8eQ}{15\pi\epsilon_0 R^3}r$, so the restoring force depends upon displacement to the first power, and we have simple harmonic motion.

(e) Comparing to $F = -kr$, $k = \dfrac{8eQ}{15\pi\epsilon_0 R^3}$. Then $\omega = \sqrt{\dfrac{k}{m_e}} = \sqrt{\dfrac{8eQ}{15\pi\epsilon_0 R^3 m_e}}$ and $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{15\pi\epsilon_0 R^3 m_e}{8eQ}}$.

EVALUATE: (f) If the amplitude of oscillation is greater than $R/2$, the force is no longer linear in r, and is thus no longer simple harmonic.

22.67. IDENTIFY: The charge in a spherical shell of radius r and thickness dr is $dQ = \rho(r)4\pi r^2\,dr$. Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius r. Let Q_i be the charge in the region $r \le R/2$ and let Q_0 be the charge in the region where $R/2 \le r \le R$.

EXECUTE: (a) The total charge is $Q = Q_i + Q_0$, where $Q_i = 4\pi\displaystyle\int_0^{R/2}\frac{3\alpha r^3}{2R}dr = \frac{6\pi\alpha}{R}\frac{1}{4}\frac{R^4}{16} = \frac{3}{32}\pi\alpha R^3$ and

$$Q_0 = 4\pi\alpha\int_{R/2}^{R}(1-(r/R)^2)r^2\,dr = 4\pi\alpha R^3\left(\frac{7}{24} - \frac{31}{160}\right) = \frac{47}{120}\pi\alpha R^3.$$ Therefore, $Q = \left(\dfrac{3}{32} + \dfrac{47}{120}\right)\pi\alpha R^3 = \dfrac{233}{480}\pi\alpha R^3$ and $\alpha = \dfrac{480Q}{233\pi R^3}$.

(b) For $r \le R/2$, Gauss's law gives $E4\pi r^2 = \dfrac{4\pi}{\epsilon_0}\displaystyle\int_0^r \frac{3\alpha r'^3}{2R}dr' = \frac{3\pi\alpha r^4}{2\epsilon_0 R}$ and $E = \dfrac{6\alpha r^2}{16\epsilon_0 R} = \dfrac{180Qr^2}{233\pi\epsilon_0 R^4}$. For $R/2 \le r \le R$,

$$E4\pi r^2 = \frac{Q_i}{\epsilon_0} + \frac{4\pi\alpha}{\epsilon_0}\int_{R/2}^{r}(1-(r'/R)^2)r'^2\,dr' = \frac{Q_i}{\epsilon_0} + \frac{4\pi\alpha}{\epsilon_0}\left(\frac{r^3}{3} - \frac{R^3}{24} - \frac{r^5}{5R^2} + \frac{R^3}{160}\right).$$

$$E4\pi r^2 = \frac{3}{128}\frac{4\pi\alpha R^3}{\epsilon_0} + \frac{4\pi\alpha R^3}{\epsilon_0}\left(\frac{1}{3}\left(\frac{r}{R}\right)^3 - \frac{1}{5}\left(\frac{r}{R}\right)^5 - \frac{17}{480}\right)$$ and $E = \dfrac{480Q}{233\pi\epsilon_0 r^2}\left(\dfrac{1}{3}\left(\dfrac{r}{R}\right)^3 - \dfrac{1}{5}\left(\dfrac{r}{R}\right)^5 - \dfrac{23}{1920}\right)$. For $r \ge R$,

$E = \dfrac{Q}{4\pi\epsilon_0 r^2}$, since all the charge is enclosed.

(c) The fraction of Q between $R/2 \le r \le R$ is $\dfrac{Q_0}{Q} = \dfrac{47}{120}\dfrac{480}{233} = 0.807.$

(d) $E = \dfrac{180}{233}\dfrac{Q}{4\pi\epsilon_0 R^2}$ using either of the electric field expressions above, evaluated at $r = R/2$.

EVALUATE: (e) The force an electron would feel never is proportional to $-r$ which is necessary for simple harmonic oscillations. It is oscillatory since the force is always attractive, but it has the wrong power of r to be simple harmonic motion.