MAGNETIC FIELD AND MAGNETIC FORCES

27

27.1. **IDENTIFY and SET UP:** Apply Eq.(27.2) to calculate $\vec{F}$. Use the cross products of unit vectors from Section 1.10.

EXECUTE: $\vec{v} = (+4.19\times10^{4}\ \text{m/s})\hat{i} + (-3.85\times10^{4}\ \text{m/s})\hat{j}$

(a) $\vec{B} = (1.40\ \text{T})\hat{i}$

$\vec{F} = q\vec{v}\times\vec{B} = (-1.24\times10^{-8}\ \text{C})(1.40\ \text{T})\left[(4.19\times10^{4}\ \text{m/s})\hat{i}\times\hat{i} - (3.85\times10^{4}\ \text{m/s})\hat{j}\times\hat{i}\right]$

$\hat{i}\times\hat{i} = 0, \hat{j}\times\hat{i} = -\hat{k}$

$\vec{F} = (-1.24\times10^{-8}\ \text{C})(1.40\ \text{T})(-3.85\times10^{4}\ \text{m/s})(-\hat{k}) = (-6.68\times10^{-4}\ \text{N})\hat{k}$

EVALUATE: The directions of $\vec{v}$ and $\vec{B}$ are shown in Figure 27.1a.

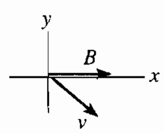


Figure 27.1a

The right-hand rule gives that $\vec{v}\times\vec{B}$ is directed out of the paper (+z-direction). The charge is negative so $\vec{F}$ is opposite to $\vec{v}\times\vec{B}$;

$\vec{F}$ is in the $-z$- direction. This agrees with the direction calculated with unit vectors.

(b) EXECUTE: $\vec{B} = (1.40\ \text{T})\hat{k}$

$\vec{F} = q\vec{v}\times\vec{B} = (-1.24\times10^{-8}\ \text{C})(1.40\ \text{T})\left[(+4.19\times10^{4}\ \text{m/s})\hat{i}\times\hat{k} - (3.85\times10^{4}\ \text{m/s})\hat{j}\times\hat{k}\right]$

$\hat{i}\times\hat{k} = -\hat{j}, \hat{j}\times\hat{k} = \hat{i}$

$\vec{F} = (-7.27\times10^{-4}\ \text{N})(-\hat{j}) + (6.68\times10^{-4}\ \text{N})\hat{i} = \left[(6.68\times10^{-4}\ \text{N})\hat{i} + (7.27\times10^{-4}\ \text{N})\hat{j}\right]$

EVALUATE: The directions of $\vec{v}$ and $\vec{B}$ are shown in Figure 27.1b.

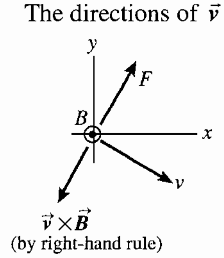


Figure 27.1b

The direction of $\vec{F}$ is opposite to $\vec{v}\times\vec{B}$ since q is negative. The direction of $\vec{F}$ computed from the right-hand rule agrees qualitatively with the direction calculated with unit vectors.

27.2. **IDENTIFY:** The net force must be zero, so the magnetic and gravity forces must be equal in magnitude and opposite in direction.

SET UP: The gravity force is downward so the force from the magnetic field must be upward. The charge's velocity and the forces are shown in Figure 27.2. Since the charge is negative, the magnetic force is opposite to the right-hand rule direction. The minimum magnetic field is when the field is perpendicular to $\vec{v}$. The force is also perpendicular to $\vec{B}$, so $\vec{B}$ is either eastward or westward.

EXECUTE: If $\vec{B}$ is eastward, the right-hand rule direction is into the page and $\vec{F}_B$ is out of the page, as required.

Therefore, $\vec{B}$ is eastward. $mg = |q|vB\sin\phi$. $\phi = 90°$ and $B = \dfrac{mg}{v|q|} = \dfrac{(0.195\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)}{(4.00\times10^{4}\ \text{m/s})(2.50\times10^{-8}\ \text{C})} = 1.91\ \text{T}$.

EVALUATE: The magnetic field could also have a component along the north-south direction, that would not contribute to the force, but then the field wouldn't have minimum magnitude.

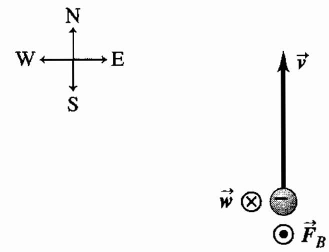


Figure 27.2

27.3. **IDENTIFY:** The force $\vec{F}$ on the particle is in the direction of the deflection of the particle. Apply the right-hand rule to the directions of $\vec{v}$ and $\vec{B}$. See if your thumb is in the direction of $\vec{F}$, or opposite to that direction. Use $F = |q|vB\sin\phi$ with $\phi = 90°$ to calculate *F*.

SET UP: The directions of $\vec{v}$, $\vec{B}$ and $\vec{F}$ are shown in Figure 27.3.

EXECUTE: **(a)** When you apply the right-hand rule to $\vec{v}$ and $\vec{B}$, your thumb points east. $\vec{F}$ is in this direction, so the charge is positive.

(b) $F = |q|vB\sin\phi = (8.50\times10^{-6}\text{ C})(4.75\times10^{3}\text{ m/s})(1.25\text{ T})\sin 90° = 0.0505\text{ N}$

EVALUATE: If the particle had negative charge and $\vec{v}$ and $\vec{B}$ are unchanged, the particle would be deflected toward the west.

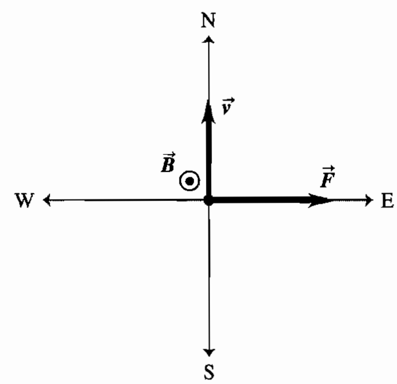


Figure 27.3

27.4. **IDENTIFY:** Apply Newton's second law, with the force being the magnetic force.

SET UP: $\hat{j}\times\hat{i} = -\hat{k}$

EXECUTE: $\vec{F} = m\vec{a} = q\vec{v}\times\vec{B}$ gives $\vec{a} = \dfrac{q\vec{v}\times\vec{B}}{m}$ and

$$\vec{a} = \frac{(1.22\times10^{-8}\text{ C})(3.0\times10^{4}\text{ m/s})(1.63\text{ T})(\hat{j}\times\hat{i})}{1.81\times10^{-3}\text{ kg}} = -(0.330\text{ m/s}^2)\hat{k}.$$

EVALUATE: The acceleration is in the $-z$-direction and is perpendicular to both $\vec{v}$ and $\vec{B}$.

27.5. **IDENTIFY:** Apply $F = |q|vB\sin\phi$ and solve for *v*.

SET UP: An electron has $q = -1.60\times10^{-19}$ C.

EXECUTE: $v = \dfrac{F}{|q|B\sin\phi} = \dfrac{4.60\times10^{-15}\text{ N}}{(1.6\times10^{-19}\text{ C})(3.5\times10^{-3}\text{ T})\sin 60°} = 9.49\times10^{6}\text{ m/s}$

EVALUATE: Only the component $B\sin\phi$ of the magnetic field perpendicular to the velocity contributes to the force.

27.6. **IDENTIFY:** Apply Newton's second law and $F = |q|vB\sin\phi$.

SET UP: ϕ is the angle between the direction of $\vec{v}$ and the direction of $\vec{B}$.

EXECUTE: **(a)** The smallest possible acceleration is zero, when the motion is parallel to the magnetic field. The greatest acceleration is when the velocity and magnetic field are at right angles:

$$a = \frac{qvB}{m} = \frac{(1.6\times10^{-19}\text{ C})(2.50\times10^{6}\text{ m/s})(7.4\times10^{-2}\text{ T})}{(9.11\times10^{-31}\text{ kg})} = 3.25\times10^{16}\text{ m/s}^2.$$

(b) If $a = \frac{1}{4}(3.25\times10^{16}\text{ m/s}^2) = \dfrac{qvB\sin\phi}{m}$, then $\sin\phi = 0.25$ and $\phi = 14.5°$.

EVALUATE: The force and acceleration decrease as the angle ϕ approaches zero.

27.7. **IDENTIFY:** Apply $\vec{F} = q\vec{v}\times\vec{B}$.

SET UP: $\vec{v} = v_y\hat{j}$, with $v_y = -3.80\times10^3\text{ m/s}$. $F_x = +7.60\times10^{-3}$ N, $F_y = 0$, and $F_z = -5.20\times10^{-3}$ N.

EXECUTE: **(a)** $F_x = q(v_yB_z - v_zB_y) = qv_yB_z$.

$B_z = F_x/qv_y = (7.60\times10^{-3}\text{ N})/([7.80\times10^{-6}\text{ C})(-3.80\times10^{3}\text{ m/s})] = -0.256\text{ T}$

$F_y = q(v_zB_x - v_xB_z) = 0$, which is consistent with $\vec{F}$ as given in the problem. There is no force component along the direction of the velocity.

$F_z = q(v_xB_y - v_yB_x) = -qv_yB_x$. $B_x = -F_z/qv_y = -0.175\text{ T}$.

(b) B_y is not determined. No force due to this component of $\vec{B}$ along $\vec{v}$; measurement of the force tells us nothing about B_y.

(c) $\vec{B}\cdot\vec{F} = B_xF_x + B_yF_y + B_zF_z = (-0.175\text{ T})(+7.60\times10^{-3}\text{ N}) + (-0.256\text{ T})(-5.20\times10^{-3}\text{ N})$

$\vec{B}\cdot\vec{F} = 0$. $\vec{B}$ and $\vec{F}$ are perpendicular (angle is $90°$).

EVALUATE: The force is perpendicular to both $\vec{v}$ and $\vec{B}$, so $\vec{v}\cdot\vec{F}$ is also zero.

27.8. **IDENTIFY and SET UP:** $\vec{F} = q\vec{v}\times\vec{B} = qB_z[v_x(\hat{i}\times\hat{k}) + v_y(\hat{j}\times\hat{k}) + v_z(\hat{k}\times\hat{k})] = qB_z[v_x(-\hat{j}) + v_y(\hat{i})]$.

EXECUTE: **(a)** Set the expression for $\vec{F}$ equal to the given value of $\vec{F}$ to obtain:

$$v_x = \frac{F_y}{-qB_z} = \frac{(7.40\times10^{-7}\text{ N})}{-(-5.60\times10^{-9}\text{ C})(-1.25\text{ T})} = -106\text{ m/s}$$

$$v_y = \frac{F_x}{qB_z} = \frac{-(3.40\times10^{-7}\text{ N})}{(-5.60\times10^{-9}\text{ C})(-1.25\text{ T})} = -48.6\text{ m/s}.$$

(b) v_z does not contribute to the force, so is not determined by a measurement of $\vec{F}$.

(c) $\vec{v}\cdot\vec{F} = v_xF_x + v_yF_y + v_zF_z = \dfrac{F_y}{-qB_z}F_x + \dfrac{F_x}{qB_z}F_y = 0$; $\theta = 90°$.

EVALUATE: The force is perpendicular to both $\vec{v}$ and $\vec{B}$, so $\vec{B}\cdot\vec{F}$ is also zero.

27.9. **IDENTIFY:** Apply $\vec{F} = q\vec{v}\times\vec{B}$ to the force on the proton and to the force on the electron. Solve for the components of $\vec{B}$.

SET UP: $\vec{F}$ is perpendicular to both $\vec{v}$ and $\vec{B}$. Since the force on the proton is in the +*y*-direction, $B_y = 0$ and $\vec{B} = B_x\hat{i} + B_z\hat{k}$. For the proton, $\vec{v} = (1.50\text{ km/s})\hat{i}$.

EXECUTE: **(a)** For the proton, $\vec{F} = q(1.50\times10^3\text{ m/s})\hat{i}\times(B_x\hat{i} + B_z\hat{k}) = q(1.50\times10^3\text{ m/s})B_z(-\hat{j})$. $\vec{F} = (2.25\times10^{-16}\text{ N})\hat{j}$,

so $B_z = -\dfrac{2.25\times10^{-16}\text{ N}}{(1.60\times10^{-19}\text{ C})(1.50\times10^3\text{ m/s})} = -0.938\text{ T}$. The force on the proton is independent of B_x. For the electron, $\vec{v} = (4.75\text{ km/s})(-\hat{k})$. $\vec{F} = q\vec{v}\times\vec{B} = (-e)(4.75\times10^3\text{ m/s})(-\hat{k})\times(B_x\hat{i} + B_z\hat{k}) = +e(4.75\times10^3\text{ m/s})B_x\hat{j}$.

The magnitude of the force is $F = e(4.75\times10^3\text{ m/s})|B_x|$. Since $F = 8.50\times10^{-16}$ N,

$|B_x| = \dfrac{8.50\times10^{-16}\text{ N}}{(1.60\times10^{-19}\text{ C})(4.75\times10^3\text{ m/s})} = 1.12\text{ T}$. $B_x = \pm1.12$ T. The sign of B_x is not determined by measuring the magnitude of the force on the electron. $B = \sqrt{B_x^2 + B_z^2} = \sqrt{(\pm1.12\text{ T}) + (-0.938\text{ T})^2} = 1.46\text{ T}$.

$\tan\theta = \dfrac{B_z}{B_x} = \dfrac{-0.938\text{ T}}{\pm1.12\text{ T}}$. $\theta = \pm40°$. $\vec{B}$ is in the *xz*-plane and is either at $40°$ from the +*x*-direction toward the $-z$-direction or $40°$ from the $-x$-direction toward the $-z$-direction.

(b) $\vec{B} = B_x\hat{i} + B_z\hat{k}$. $\vec{v}$ = (3.2 km/s)$(-\hat{j})$.

$\vec{F} = q\vec{v}\times\vec{B} = (-e)(3.2\text{ km/s})(-\hat{j})\times(B_x\hat{i} + B_z\hat{k}) = e(3.2\times10^3\text{ m/s})(B_x(-\hat{k}) + B_z\hat{i})$.

$\vec{F} = e(3.2\times10^3\text{ m/s})(-[\pm1.12\text{ T}]\hat{k} - [0.938\text{ T}]\hat{i}) = -(4.80\times10^{-16}\text{ N})\hat{i} \pm (5.73\times10^{-16}\text{ N})\hat{k}$

$F = \sqrt{F_x^2 + F_z^2} = 7.47\times10^{-16}$ N . $\tan\theta = \dfrac{F_z}{F_x} = \dfrac{\pm5.73\times10^{-16}\text{ N}}{-4.80\times10^{-16}\text{ N}}$. $\theta = \pm50.0°$. The force is in the xz-plane and is directed at 50.0° from the $-x$-axis toward either the $+z$ or $-z$ axis, depending on the sign of B_x .

EVALUATE: If the direction of the force on the first electron were measured, then the sign of B_x would be determined.

27.10. **IDENTIFY:** Magnetic field lines are closed loops, so the net flux through any closed surface is zero.
SET UP: Let magnetic field directed out of the enclosed volume correspond to positive flux and magnetic field directed into the volume correspond to negative flux.
EXECUTE: **(a)** The total flux must be zero, so the flux through the remaining surfaces must be –0.120 Wb.
(b) The shape of the surface is unimportant, just that it is closed.
(c) One possibility is sketched in Figure 27.10.
EVALUATE: In Figure 27.10 all the field lines that enter the cube also exit through the surface of the cube.

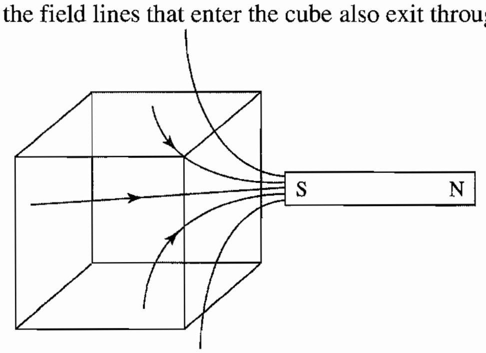


Figure 27.10

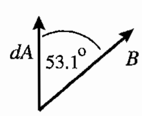
27.11. **IDENTIFY** and **SET UP:** $\Phi_B = \int\vec{B}\cdot d\vec{A}$

Circular area in the xy-plane, so $A = \pi r^2 = \pi(0.0650\text{ m})^2 = 0.01327\text{ m}^2$ and $d\vec{A}$ is in the z-direction. Use Eq.(1.18) to calculate the scalar product.

EXECUTE: **(a)** $\vec{B} = (0.230\text{ T})\hat{k}$; $\vec{B}$ and $d\vec{A}$ are parallel $(\phi = 0°)$ so $\vec{B}\cdot d\vec{A} = B\,dA$.

B is constant over the circular area so $\Phi_B = \int\vec{B}\cdot d\vec{A} = \int B\,dA = B\int dA = BA = (0.230\text{ T})(0.01327\text{ m}^2) = 3.05\times10^{-3}$ Wb

(b) The directions of $\vec{B}$ and $d\vec{A}$ are shown in Figure 27.11a.



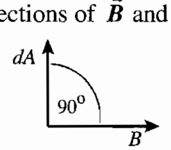
$\vec{B}\cdot d\vec{A} = B\cos\phi\, dA$
with $\phi = 53.1°$

Figure 27.11a

B and ϕ are constant over the circular area so $\Phi_B = \int\vec{B}\cdot d\vec{A} = \int B\cos\phi\, dA = B\cos\phi\int dA = B\cos\phi A$

$\Phi_B = (0.230\text{ T})\cos53.1°(0.01327\text{ m}^2) = 1.83\times10^{-3}$ Wb

(c) The directions of $\vec{B}$ and $d\vec{A}$ are shown in Figure 27.11b.



$\vec{B}\cdot d\vec{A} = 0$ since $d\vec{A}$ and $\vec{B}$ are perpendicular $(\phi = 90°)$

$\Phi_B = \int\vec{B}\cdot d\vec{A} = 0.$

Figure 27.11b

EVALUATE: Magnetic flux is a measure of how many magnetic field lines pass through the surface. It is maximum when $\vec{B}$ is perpendicular to the plane of the loop (part a) and is zero when $\vec{B}$ is parallel to the plane of the loop (part c).

27.12. **IDENTIFY:** When $\vec{B}$ is uniform across the surface, $\Phi_B = \vec{B}\cdot\vec{A} = BA\cos\phi$.

SET UP: $\vec{A}$ is normal to the surface and is directed outward from the enclosed volume. For surface *abcd*, $\vec{A} = -A\hat{i}$. For surface *befc*, $\vec{A} = -A\hat{k}$. For surface *aefd*, $\cos\phi = 3/5$ and the flux is positive.

EXECUTE: **(a)** $\Phi_B(abcd) = \vec{B}\cdot\vec{A} = 0.$

(b) $\Phi_B(befc) = \vec{B}\cdot\vec{A} = -(0.128\text{ T})(0.300\text{ m})(0.300\text{ m}) = -0.0115\text{ Wb}.$

(c) $\Phi_B(aefd) = \vec{B}\cdot\vec{A} = BA\cos\phi = \frac{3}{5}(0.128\text{ T})(0.500\text{ m})(0.300\text{ m}) = +0.0115\text{ Wb}.$

(d) The net flux through the rest of the surfaces is zero since they are parallel to the x-axis. The total flux is the sum of all parts above, which is zero.

EVALUATE: The total flux through any closed surface, that encloses a volume, is zero.

27.13. **IDENTIFY:** The total flux through the bottle is zero because it is a closed surface.

SET UP: The total flux through the bottle is the flux through the plastic plus the flux through the open cap, so the sum of these must be zero. $\Phi_{\text{plastic}} + \Phi_{\text{cap}} = 0.$

$\Phi_{\text{plastic}} = -\Phi_{\text{cap}} = -BA\cos\varphi = -B(\pi r^2)\cos\varphi$

EXECUTE: Substituting the numbers gives $\Phi_{\text{plastic}} = -(1.75\text{ T})\pi(0.0125\text{ m})^2\cos 25° = -7.8\times 10^{-4}\text{ Wb}$

EVALUATE: It would be impossible to calculate the flux through the plastic directly because of the complicated shape of the bottle, but with a little thought we can find this flux through a simple calculation.

27.14. **IDENTIFY:** $p = mv$ and $L = Rp$, since the velocity and linear momentum are tangent to the circular path.

SET UP: $|q|vB = mv^2/R$.

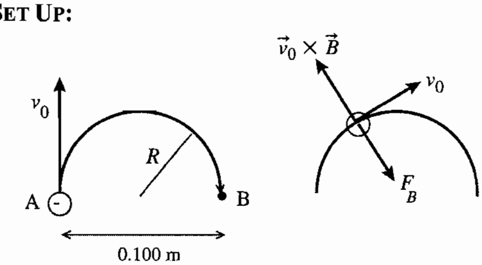
EXECUTE: **(a)** $p = mv = m\left(\dfrac{RqB}{m}\right) = RqB = (4.68\times10^{-3}\text{ m})(6.4\times10^{-19}\text{ C})(1.65\text{ T}) = 4.94\times10^{-21}\text{ kg m/s}.$

(b) $L = Rp = R^2qB = (4.68\times10^{-3}\text{ m})^2(6.4\times10^{-19}\text{ C})(1.65\text{ T}) = 2.31\times10^{-23}\text{ kg}\cdot\text{m}^2/\text{s}.$

EVALUATE: $\vec{p}$ is tangent to the orbit and $\vec{L}$ is perpendicular to the orbit plane.

27.15. **(a) IDENTIFY:** Apply Eq.(27.2) to relate the magnetic force $\vec{F}$ to the directions of $\vec{v}$ and $\vec{B}$. The electron has negative charge so $\vec{F}$ is opposite to the direction of $\vec{v}\times\vec{B}$. For motion in an arc of a circle the acceleration is toward the center of the arc so $\vec{F}$ must be in this direction. $a = v^2/R$.

SET UP:



As the electron moves in the semicircle, its velocity is tangent to the circular path. The direction of $\vec{v}_0\times\vec{B}$ at a point along the path is shown in Figure 27.15.

Figure 27.15

EXECUTE: For circular motion the acceleration of the electron $\vec{a}_{\text{rad}}$ is directed in toward the center of the circle. Thus the force $\vec{F}_B$ exerted by the magnetic field, since it is the only force on the electron, must be radially inward. Since q is negative, $\vec{F}_B$ is opposite to the direction given by the right-hand rule for $\vec{v}_0\times\vec{B}$. Thus $\vec{B}$ is directed into the page. Apply Newton's 2nd law to calculate the magnitude of $\vec{B}$: $\sum\vec{F} = m\vec{a}$ gives $\sum F_{\text{rad}} = ma$

$F_B = m(v^2/R)$

$F_B = |q|vB\sin\phi = |q|vB$, so $|q|vB = m(v^2/R)$

$$B = \frac{mv}{|q|R} = \frac{(9.109\times10^{-31}\text{ kg})(1.41\times10^{6}\text{ m/s})}{(1.602\times10^{-19}\text{ C})(0.050\text{ m})} = 1.60\times10^{-4}\text{ T}$$

(b) IDENTIFY and SET UP: The speed of the electron as it moves along the path is constant. ($\vec{F}_B$ changes the direction of $\vec{v}$ but not its magnitude.) The time is given by the distance divided by v_0.

EXECUTE: The distance along the semicircular path is πR, so $t = \dfrac{\pi R}{v_0} = \dfrac{\pi(0.050\text{ m})}{1.41\times10^{6}\text{ m/s}} = 1.11\times10^{-7}\text{ s}$

EVALUATE: The magnetic field required increases when v increases or R decreases and also depends on the mass to charge ratio of the particle.

27.16. **IDENTIFY:** Newton's second law gives $|q|vB = mv^2/R$. The speed v is constant and equals v_0. The direction of the magnetic force must be in the direction of the acceleration and is toward the center of the semicircular path.

SET UP: A proton has $q = +1.60\times10^{-19}$ C and $m = 1.67\times10^{-27}$ kg. The direction of the magnetic force is given by the right-hand rule.

EXECUTE: **(a)** $B=\dfrac{mv}{qR}=\dfrac{(1.67\times 10^{-27}\text{ kg})(1.41\times 10^{6}\text{ m/s})}{(1.60\times\ 10^{-19}\text{ C})(0.0500\text{ m})}=0.294\text{ T}$

The direction of the magnetic field is out of the page (the charge is positive), in order for $\vec{F}$ to be directed to the right at point A.

(b) The time to complete half a circle is $t=\pi R/v_0=1.11\times 10^{-7}$ s.

EVALUATE: The magnetic field required to produce this path for a proton has a different magnitude (because of the different mass) and opposite direction (because of opposite sign of the charge) than the field required to produce the path for an electron.

27.17. **IDENTIFY** and **SET UP:** Use conservation of energy to find the speed of the ball when it reaches the bottom of the shaft. The right-hand rule gives the direction of $\vec{F}$ and Eq.(27.1) gives its magnitude. The number of excess electrons determines the charge of the ball.

EXECUTE: $q=(4.00\times 10^{8})(-1.602\times 10^{-19}\text{ C})=-6.408\times 10^{-11}\text{ C}$

speed at bottom of shaft: $\frac{1}{2}mv^2=mgy$; $v=\sqrt{2gy}=49.5$ m/s

$\vec{v}$ is downward and $\vec{B}$ is west, so $\vec{v}\times\vec{B}$ is north. Since $q<0$, $\vec{F}$ is south.

$F=|q|vB\sin\theta=(6.408\times 10^{-11}\text{ C})(49.5\text{ m/s})(0.250\text{ T})\sin 90°=7.93\times 10^{-10}\text{ N}$

EVALUATE: Both the charge and speed of the ball are relatively small so the magnetic force is small, much less than the gravity force of 1.5 N.

27.18. **IDENTIFY:** Since the particle moves perpendicular to the uniform magnetic field, the radius of its path is $R=\dfrac{mv}{|q|B}$. The magnetic force is perpendicular to both $\vec{v}$ and $\vec{B}$.

SET UP: The alpha particle has charge $q=+2e=3.20\times 10^{-19}$ C.

EXECUTE: **(a)** $R=\dfrac{(6.64\times 10^{-27}\text{ kg})(35.6\times 10^{3}\text{ m/s})}{(3.20\times 10^{-19}\text{ C})(1.10\text{ T})}=6.73\times 10^{-4}\text{ m}=0.673\text{ mm}$. The alpha particle moves in a circular arc of diameter $2R=1.35$ mm.

(b) For a very short time interval the displacement of the particle is in the direction of the velocity. The magnetic force is always perpendicular to this direction so it does no work. The work-energy theorem therefore says that the kinetic energy of the particle, and hence its speed, is constant.

(c) The acceleration is $a=\dfrac{F_B}{m}=\dfrac{|q|vB\sin\phi}{m}=\dfrac{(3.20\times 10^{-19}\text{ C})(35.6\times 10^{3}\text{ m/s})(1.10\text{ T})\sin 90°}{6.64\times 10^{-27}\text{ kg}}=1.88\times 10^{12}\text{ m/s}^2$. We can also use $a=\dfrac{v^2}{R}$ and the result of part (a) to calculate $a=\dfrac{(35.6\times 10^{3}\text{ m/s})^2}{6.73\times 10^{-4}\text{ m}}=1.88\times 10^{12}\text{ m/s}^2$, the same result. The acceleration is perpendicular to $\vec{v}$ and $\vec{B}$ and so is horizontal, toward the center of curvature of the particle's path.

EVALUATE: **(d)** The unbalanced force ($\vec{F}_B$) is perpendicular to $\vec{v}$, so it changes the direction of $\vec{v}$ but not its magnitude, which is the speed.

27.19. **IDENTIFY:** In part (a), apply conservation of energy to the motion of the two nuclei. In part (b) apply $|q|vB=mv^2/R$.

SET UP: In part (a), let point 1 be when the two nuclei are far apart and let point 2 be when they are at their closest separation.

EXECUTE: **(a)** $K_1+U_1=K_2+U_2$. $U_1=K_2=0$, so $K_1=U_2$ and $\frac{1}{2}mv^2=ke^2/r$.

$$v=e\sqrt{\frac{2k}{mr}}=(1.602\times 10^{-19}\text{ C})\sqrt{\frac{2k}{(3.34\times 10^{-27}\text{ kg})(1.0\times 10^{-15}\text{ m})}}=1.2\times 10^{7}\text{ m/s}$$

(b) $\sum\vec{F}=m\vec{a}$ gives $qvB=mv^2/r$. $B=\dfrac{mv}{qr}=\dfrac{(3.34\times 10^{-27}\text{ kg})(1.2\times 10^{7}\text{m/s})}{(1.602\times 10^{-19}\text{ C})(2.50\text{ m})}=0.10\text{ T}$.

EVALUATE: The speed calculated in part (a) is large, 4% of the speed of light.

27.20. **IDENTIFY:** $F=|q|vB\sin\phi$. The direction of $\vec{F}$ is given by the right-hand rule.

SET UP: An electron has $q=-e$.

EXECUTE: **(a)** $F=|q|vB\sin\phi$. $B=\dfrac{F}{|q|v\sin\phi}=\dfrac{0.00320\times 10^{-9}\text{ N}}{8(1.60\times 10^{-19}\text{ C})(500{,}000\text{ m/s})\sin 90°}=5.00\text{ T}$. If the angle ϕ is less than 90°, a larger field is needed to produce the same force. The direction of the field must be toward the south so that $\vec{v}\times\vec{B}$ is downward.

(b) $F = |q|vB\sin\phi$. $v = \dfrac{F}{|q|B\sin\phi} = \dfrac{4.60\times10^{-12}\text{ N}}{(1.60\times10^{-19}\text{ C})(2.10\text{ T})\sin 90^\circ} = 1.37\times10^{7}\text{ m/s}$. If ϕ is less than 90°, the speed would have to be larger to have the same force. The force is upward, so $\vec{v}\times\vec{B}$ must be downward since the electron is negative, and the velocity must be toward the south.

EVALUATE: The component of $\vec{B}$ along the direction of $\vec{v}$ produces no force and the component of $\vec{v}$ along the direction of $\vec{B}$ produces no force.

27.21. **(a) IDENTIFY** and **SET UP:** Apply Newton's 2nd law, with $a = v^2/R$ since the path of the particle is circular.

EXECUTE: $\sum\vec{F} = m\vec{a}$ says $|q|vB = m\left(v^2/R\right)$

$$v = \frac{|q|BR}{m} = \frac{\left(1.602\times10^{-19}\text{ C}\right)\left(2.50\text{ T}\right)\left(6.96\times10^{-3}\text{ m}\right)}{3.34\times10^{-27}\text{ kg}} = 8.35\times10^{5}\text{ m/s}$$

(b) IDENTIFY and **SET UP:** The speed is constant so t = distance/v.

EXECUTE: $t = \dfrac{\pi R}{v} = \dfrac{\pi\left(6.96\times10^{-3}\text{ m}\right)}{8.35\times10^{5}\text{ m/s}} = 2.62\times10^{-8}\text{ s}$

(c) IDENTIFY and **SET UP:** kinetic energy gained = electric potential energy lost

EXECUTE: $\tfrac{1}{2}mv^2 = |q|V$

$$V = \frac{mv^2}{2|q|} = \frac{\left(3.34\times10^{-27}\text{ kg}\right)\left(8.35\times10^{5}\text{ m/s}\right)^2}{2\left(1.602\times10^{-19}\text{ C}\right)} = 7.27\times10^{3}\text{ V} = 7.27\text{ kV}$$

EVALUATE: The deutron has a much larger mass to charge ratio than an electron so a much larger B is required for the same v and R. The deutron has positive charge so gains kinetic energy when it goes from high potential to low potential.

27.22. **IDENTIFY:** For motion in an arc of a circle, $a = \dfrac{v^2}{R}$ and the net force is radially inward, toward the center of the circle.

SET UP: The direction of the force is shown in Figure 27.22. The mass of a proton is 1.67×10^{-27} kg.

EXECUTE: **(a)** $\vec{F}$ is opposite to the right-hand rule direction, so the charge is negative. $\vec{F} = m\vec{a}$ gives $|q|vB\sin\phi = m\dfrac{v^2}{R}$. $\phi = 90^\circ$ and $v = \dfrac{|q|BR}{m} = \dfrac{3(1.60\times10^{-19}\text{ C})(0.250\text{ T})(0.475\text{ m})}{12(1.67\times10^{-27}\text{ kg})} = 2.84\times10^{6}\text{ m/s}$.

(b) $F_B = |q|vB\sin\phi = 3(1.60\times10^{-19}\text{ C})(2.84\times10^{6}\text{ m/s})(0.250\text{ T})\sin 90^\circ = 3.41\times10^{-13}\text{ N}$.

$w = mg = 12(1.67\times10^{-27}\text{ kg})(9.80\text{ m/s}^2) = 1.96\times10^{-25}\text{ N}$. The magnetic force is much larger than the weight of the particle, so it is a very good approximation to neglect gravity.

EVALUATE: **(c)** The magnetic force is always perpendicular to the path and does no work. The particles move with constant speed.

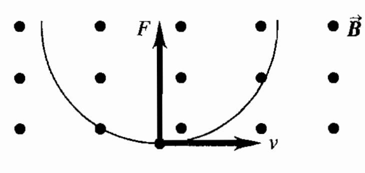


Figure 27.22

27.23. **IDENTIFY:** Example 27.3 shows that $B = \dfrac{m2\pi f}{|q|}$, where f is the frequency, in Hz, of the electromagnetic waves that are produced.

SET UP: An electron has charge $q = -e$ and mass $m = 9.11\times10^{-31}$ kg. A proton has charge $q = +e$ and mass $m = 1.67\times10^{-27}$ kg.

EXECUTE: **(a)** $B = \dfrac{m2\pi f}{|q|} = \dfrac{(9.11\times10^{-31}\text{ kg})2\pi(3.00\times10^{12}\text{ Hz})}{(1.60\times10^{-19}\text{ C})} = 107\text{ T}$. This is about 2.4 times the greatest magnitude of magnetic field yet obtained on earth.

(b) Protons have a greater mass than the electrons, so a greater magnetic field would be required to accelerate them with the same frequency and there would be no advantage in using them.

EVALUATE: Electromagnetic waves with frequency $f = 3.0$ THz have a wavelength in air of $\lambda = \dfrac{v}{f} = 1.0\times10^{-4}$ m. The shorter the wavelength the greater the frequency and the greater the magnetic field that is required. B depends only on f and on the mass-to-charge ratio of the particle that moves in the circular path.

27.24. **IDENTIFY:** The magnetic force on the beam bends it through a quarter circle.

SET UP: The distance that particles in the beam travel is $s = R\theta$, and the radius of the quarter circle is $R = mv/qB$.

EXECUTE: Solving for R gives $R = s/\theta = s/(\pi/2) = 1.18$ cm$/(\pi/2) = 0.751$ cm. Solving for the magnetic field: $B = mv/qR = (1.67\times10^{-27}\text{ kg})(1200\text{ m/s})/[(1.60\times10^{-19}\text{ C})(0.00751\text{ m})] = 1.67\times10^{-3}\text{ T}$

EVALUATE: This field is about 10 times stronger than the Earth's magnetic field, but much weaker than many laboratory fields.

27.25. **IDENTIFY:** When a particle of charge $-e$ is accelerated through a potential difference of magnitude V, it gains kinetic energy eV. When it moves in a circular path of radius R, its acceleration is $\dfrac{v^2}{R}$.

SET UP: An electron has charge $q = -e = -1.60\times10^{-19}$ C and mass 9.11×10^{-31} kg.

EXECUTE: $\frac{1}{2}mv^2 = eV$ and $v = \sqrt{\dfrac{2eV}{m}} = \sqrt{\dfrac{2(1.60\times10^{-19}\text{ C})(2.00\times10^{3}\text{ V})}{9.11\times10^{-31}\text{ kg}}} = 2.65\times10^{7}\text{ m/s}$. $\vec{F} = m\vec{a}$ gives

$|q|vB\sin\phi = m\dfrac{v^2}{R}$. $\phi = 90°$ and $B = \dfrac{mv}{|q|R} = \dfrac{(9.11\times10^{-31}\text{ kg})(2.65\times10^{7}\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.180\text{ m})} = 8.38\times10^{-4}\text{ T}$.

EVALUATE: The smaller the radius of the circular path, the larger the magnitude of the magnetic field that is required.

27.26. **IDENTIFY:** After being accelerated through a potential difference V the ion has kinetic energy qV. The acceleration in the circular path is v^2/R.

SET UP: The ion has charge $q = +e$.

EXECUTE: $K = qV = +eV$. $\frac{1}{2}mv^2 = eV$ and $v = \sqrt{\dfrac{2eV}{m}} = \sqrt{\dfrac{2(1.60\times10^{-19}\text{ C})(220\text{ V})}{1.16\times10^{-26}\text{ kg}}} = 7.79\times10^{4}$ m/s. $F_B = |q|vB\sin\phi$.

$\phi = 90°$. $\vec{F} = m\vec{a}$ gives $|q|vB = m\dfrac{v^2}{R}$. $R = \dfrac{mv}{|q|B} = \dfrac{(1.16\times10^{-26}\text{ kg})(7.79\times10^{4}\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.723\text{ T})} = 7.81\times10^{-3}\text{ m} = 7.81\text{ mm}$.

EVALUATE: The larger the accelerating voltage, the larger the speed of the particle and the larger the radius of its path in the magnetic field.

27.27. **(a) IDENTIFY** and **SET UP:** Eq.(27.4) gives the total force on the proton. At $t = 0$,

$\vec{F} = q\vec{v}\times\vec{B} = q\left(v_x\hat{i} + v_z\hat{k}\right)\times B_x\hat{i} = qv_zB_x\hat{j}$. $\vec{F} = \left(1.60\times10^{-19}\text{ C}\right)\left(2.00\times10^{5}\text{ m/s}\right)\left(0.500\text{ T}\right)\hat{j} = \left(1.60\times10^{-14}\text{ N}\right)\hat{j}$.

(b) Yes. The electric field exerts a force in the direction of the electric field, since the charge of the proton is positive, and there is a component of acceleration in this direction.

(c) EXECUTE: In the plane perpendicular to $\vec{B}$ (the yz-plane) the motion is circular. But there is a velocity component in the direction of $\vec{B}$, so the motion is a helix. The electric field in the $+\hat{i}$ direction exerts a force in the $+\hat{i}$ direction. This force produces an acceleration in the $+\hat{i}$ direction and this causes the pitch of the helix to vary. The force does not affect the circular motion in the yz-plane, so the electric field does not affect the radius of the helix.

(d) IDENTIFY and **SET UP:** Eq.(27.12) and $T = 2\pi/\omega$ to calculate the period of the motion. Calculate a_x produced by the electric force and use a constant acceleration equation to calculate the displacement in the x-direction in time $T/2$.

EXECUTE: Calculate the period T: $\omega = |q|B/m$

$T = \dfrac{2\pi}{\omega} = \dfrac{2\pi m}{|q|B} = \dfrac{2\pi\left(1.67\times10^{-27}\text{ kg}\right)}{\left(1.60\times10^{-19}\text{ C}\right)(0.500\text{ T})} = 1.312\times10^{-7}$ s. Then $t = T/2 = 6.56\times10^{-8}$ s. $v_{0x} = 1.50\times10^{5}$ m/s

$$a_x = \frac{F_x}{m} = \frac{\left(1.60\times10^{-19}\text{ C}\right)\left(2.00\times10^{4}\text{ V/m}\right)}{1.67\times10^{-27}\text{ kg}} = +1.916\times10^{12}\text{ m/s}^2$$

$$x - x_0 = v_{0x}t + \tfrac{1}{2}a_xt^2$$

$$x - x_0 = \left(1.50\times10^{5}\text{ m/s}\right)\left(6.56\times10^{-8}\text{ s}\right) + \tfrac{1}{2}\left(1.916\times10^{12}\text{ m/s}^2\right)\left(6.56\times10^{-8}\text{ s}\right)^2 = 1.40\text{ cm}$$

EVALUATE: The electric and magnetic fields are in the same direction but produce forces that are in perpendicular directions to each other.

27.28. **IDENTIFY:** For no deflection the magnetic and electric forces must be equal in magnitude and opposite in direction.

SET UP: $v = E/B$ for no deflection. With only the magnetic force, $|q|vB = mv^2/R$

EXECUTE: **(a)** $v = E/B = (1.56\times10^4\ \text{V/m})/(4.62\times10^{-3}\ \text{T}) = 3.38\times10^6\ \text{m/s}$.

(b) The directions of the three vectors $\vec{v}$, $\vec{E}$ and $\vec{B}$ are sketched in Figure 27.28.

(c) $R = \dfrac{mv}{|q|B} = \dfrac{(9.11\times10^{-31}\ \text{kg})(3.38\times10^6\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(4.62\times10^{-3}\ \text{T})} = 4.17\times10^{-3}\ \text{m}.$

$$T = \frac{2\pi m}{|q|B} = \frac{2\pi R}{v} = \frac{2\pi(4.17\times10^{-3}\ \text{m})}{(3.38\times10^6\ \text{m/s})} = 7.74\times10^{-9}\ \text{s}.$$

EVALUATE: For the field directions shown in Figure 27.28, the electric force is toward the top of the page and the magnetic force is toward the bottom of the page.

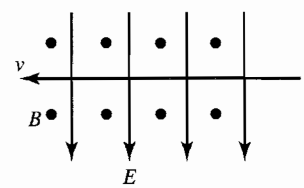


Figure 27.28

27.29. **IDENTIFY:** For the alpha particles to emerge from the plates undeflected, the magnetic force on them must exactly cancel the electric force. The battery produces an electric field between the plates, which acts on the alpha particles.

SET UP: First use energy conservation to find the speed of the alpha particles as they enter the plates: $qV = 1/2\ mv^2$. The electric field between the plates due to the battery is $E = V_b d$. For the alpha particles not to be deflected, the magnetic force must cancel the electric force, so $qvB = qE$, giving $B = E/v$.

EXECUTE: Solve for the speed of the alpha particles just as they enter the region between the plates. Their charge is $2e$.

$$v_\alpha = \sqrt{\frac{2(2e)V}{m}} = \sqrt{\frac{4(1.60\times10^{-19}\ \text{C})(1750\ \text{V})}{6.64\times10^{-27}\ \text{kg}}} = 4.11\times10^5\ \text{m/s}$$

The electric field between the plates, produced by the battery, is

$$E = V_b/d = (150\ \text{V})/(0.00820\ \text{m}) = 18{,}300\ \text{V}$$

The magnetic force must cancel the electric force:

$$B = E/v_\alpha = (18{,}300\ \text{V})/(4.11\times10^5\ \text{m/s}) = 0.0445\ \text{T}$$

The magnetic field is perpendicular to the electric field. If the charges are moving to the right and the electric field points upward, the magnetic field is out of the page.

EVALUATE: The sign of the charge of the alpha particle does not enter the problem, so negative charges of the same magnitude would also not be deflected.

27.30. **IDENTIFY:** For no deflection the magnetic and electric forces must be equal in magnitude and opposite in direction.

SET UP: $v = E/B$ for no deflection.

EXECUTE: To pass undeflected in both cases, $E = vB = (5.85\times10^3\ \text{m/s})(1.35\ \text{T}) = 7898\ \text{N/C}$.

(a) If $q = 0.640\times10^{-9}$ C, the electric field direction is given by $-(\hat{j}\times(-\hat{k})) = \hat{i}$, since it must point in the opposite direction to the magnetic force.

(b) If $q = -0.320\times10^{-9}$ C, the electric field direction is given by $((-\hat{j})\times(-\hat{k})) = \hat{i}$, since the electric force must point in the opposite direction as the magnetic force. Since the particle has negative charge, the electric force is opposite to the direction of the electric field and the magnetic force is opposite to the direction it has in part (a).

EVALUATE: The same configuration of electric and magnetic fields works as a velocity selector for both positively and negatively charged particles.

27.31. **IDENTIFY** and **SET UP:** Use the fields in the velocity selector to find the speed v of the particles that pass through. Apply Newton's 2nd law with $a = v^2/R$ to the circular motion in the second region of the spectrometer. Solve for the mass m of the ion.

EXECUTE: In the velocity selector $|q|E = |q|vB$.

$$v = \frac{E}{B} = \frac{1.12\times10^5\ \text{V/m}}{0.540\ \text{T}} = 2.074\times10^5\ \text{m/s}$$

In the region of the circular path $\sum \vec{F} = m\vec{a}$ gives $|q|vB = m(v^2/R)$ so $m = |q|RB/v$

Singly charged ion, so $|q| = +e = 1.602\times10^{-19}$ C

$$m = \frac{(1.602\times10^{-19}\text{ C})(0.310\text{ m})(0.540\text{ T})}{2.074\times10^5\text{ m/s}} = 1.29\times10^{-25}\text{ kg}$$

Mass number = mass in atomic mass units, so is $\dfrac{1.29\times10^{-25}\text{ kg}}{1.66\times10^{-27}\text{ kg}} = 78.$

EVALUATE: Appendix D gives the average atomic mass of selenium to be 78.96. One of its isotopes has atomic mass 78.

27.32. **IDENTIFY** and **SET UP:** For a velocity selector, $E = vB$. For parallel plates with opposite charge, $V = Ed$.

EXECUTE: **(a)** $E = vB = (1.82\times10^6\text{ m/s})(0.650\text{ T}) = 1.18\times10^6\text{ V/m}$.

(b) $V = Ed = (1.18\times10^6\text{ V/m})(5.20\times10^{-3}\text{ m}) = 6.14\text{ kV}$.

EVALUATE: Any charged particle with $v = 1.82\times10^6$ m/s will pass through undeflected, regardless of the sign and magnitude of its charge.

27.33. **IDENTIFY:** The magnetic force is $F = IlB\sin\phi$. For the wire to be completely supported by the field requires that $F = mg$ and that $\vec{F}$ and $\vec{w}$ are in opposite directions.

SET UP: The magnetic force is maximum when $\phi = 90°$. The gravity force is downward.

EXECUTE: **(a)** $IlB = mg$. $I = \dfrac{mg}{lB} = \dfrac{(0.150\text{ kg})(9.80\text{ m/s}^2)}{(2.00\text{ m})(0.55\times10^{-4}\text{ T})} = 1.34\times10^4$ A. This is a very large current and ohmic heating due to the resistance of the wire would be severe; such a current isn't feasible.

(b) The magnetic force must be upward. The directions of I, $\vec{B}$ and $\vec{F}$ are shown in Figure 27.33, where we have assumed that $\vec{B}$ is south to north. To produce an upward magnetic force, the current must be to the east. The wire must be horizontal and perpendicular to the earth's magnetic field.

EVALUATE: The magnetic force is perpendicular to both the direction of I and the direction of $\vec{B}$.

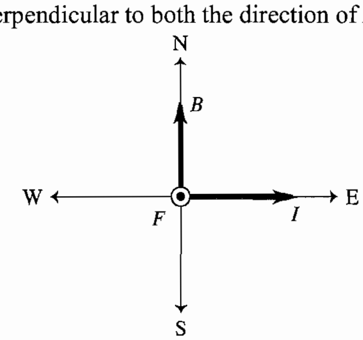


Figure 27.33

27.34. **IDENTIFY:** Apply $F = IlB\sin\phi$.

SET UP: $l = 0.0500$ m is the length of wire in the magnetic field. Since the wire is perpendicular to $\vec{B}$, $\phi = 90°$.

EXECUTE: $F = IlB = (10.8\text{ A})(0.0500\text{ m})(0.550\text{ T}) = 0.297\text{ N}$.

EVALUATE: The force per unit length of wire is proportional to both B and I.

27.35. **IDENTIFY:** Apply $F = IlB\sin\phi$.

SET UP: Label the three segments in the field as a, b, and c. Let x be the length of segment a. Segment b has length 0.300 m and segment c has length $0.600\text{ m} - x$. Figure 27.35a shows the direction of the force on each segment. For each segment, $\phi = 90°$. The total force on the wire is the vector sum of the forces on each segment.

EXECUTE: $F_a = IlB = (4.50\text{ A})x(0.240\text{ T})$. $F_c = (4.50\text{ A})(0.600\text{ m} - x)(0.240\text{ T})$. Since $\vec{F}_a$ and $\vec{F}_c$ are in the same direction their vector sum has magnitude $F_{ac} = F_a + F_c = (4.50\text{ A})(0.600\text{ m})(0.240\text{ T}) = 0.648\text{ N}$ and is directed toward the bottom of the page in Figure 27.35a. $F_b = (4.50\text{ A})(0.300\text{ m})(0.240\text{ T}) = 0.324\text{ N}$ and is directed to the right. The vector addition diagram for $\vec{F}_{ac}$ and $\vec{F}_b$ is given in Figure 27.35b.

$F = \sqrt{F_{ac}^2 + F_b^2} = \sqrt{(0.648\text{ N})^2 + (0.324\text{ N})^2} = 0.724\text{ N}$. $\tan\theta = \dfrac{F_{ac}}{F_b} = \dfrac{0.648\text{ N}}{0.324\text{ N}}$ and $\theta = 63.4°$. The net force has magnitude 0.724 N and its direction is specified by $\theta = 63.4°$ in Figure 27.35b.

EVALUATE: All three current segments are perpendicular to the magnetic field, so $\phi = 90°$ for each in the force equation. The direction of the force on a segment depends on the direction of the current for that segment.

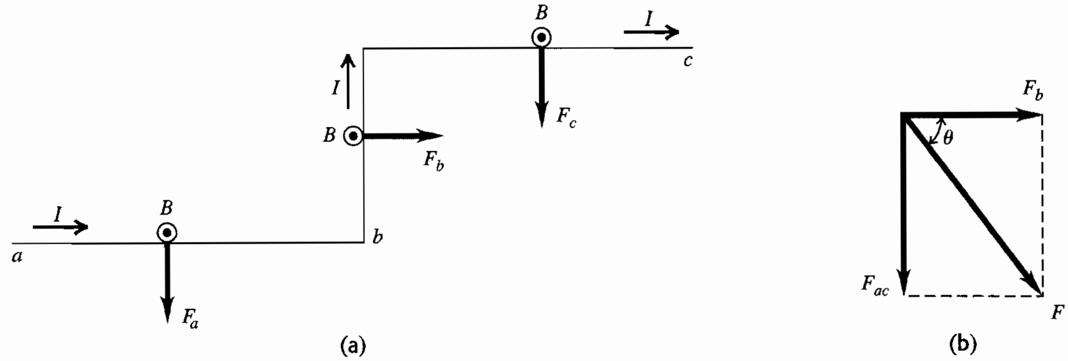


Figure 27.35

27.36. **IDENTIFY and SET UP:** $F = IlB\sin\phi$. The direction of $\vec{F}$ is given by applying the right-hand rule to the directions of I and $\vec{B}$.

EXECUTE: **(a)** The current and field directions are shown in Figure 27.36a. The right-hand rule gives that $\vec{F}$ is directed to the south, as shown. $\phi = 90°$ and $F = (1.20\text{A})(1.00\times10^{-2}\text{ m})(0.588\text{ T}) = 7.06\times10^{-3}\text{ N}$.

(b) The right-hand rule gives that $\vec{F}$ is directed to the west, as shown in Figure 27.36b. $\phi = 90°$ and $F = 7.06\times10^{-3}\text{ N}$, the same as in part (a).

(c) The current and field directions are shown in Figure 27.36c. The right-hand rule gives that $\vec{F}$ is 60.0° north of west. $\phi = 90°$ so $F = 7.06\times10^{-3}\text{ N}$, the same as in part (a).

EVALUATE: In each case the current direction is perpendicular to the magnetic field. The magnitude of the magnetic force is the same in each case but its direction depends on the direction of the magnetic field.

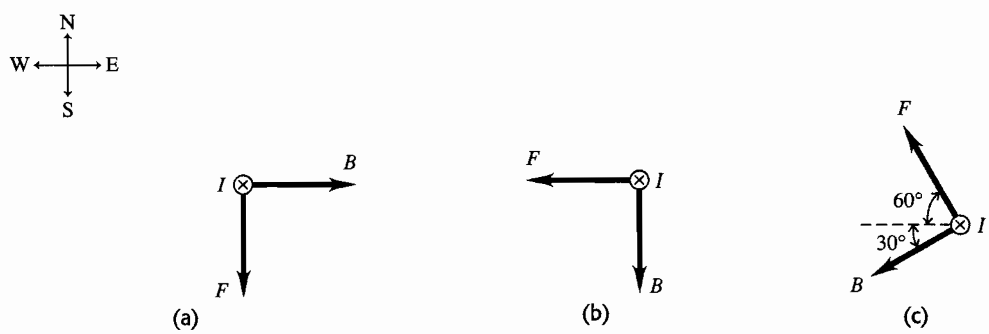


Figure 27.36

27.37. **IDENTIFY:** $F = IlB\sin\phi$.

SET UP: Since the field is perpendicular to the rod it is perpendicular to the current and $\phi = 90°$.

EXECUTE: $I = \dfrac{F}{lB} = \dfrac{0.13\text{ N}}{(0.200\text{ m})(0.067\text{ T})} = 9.7\text{ A}$

EVALUATE: The force and current are proportional. We have assumed that the entire 0.200 m length of the rod is in the magnetic field.

27.38. **IDENTIFY:** Apply $\vec{F} = I\vec{l}\times\vec{B}$.

SET UP: The magnetic field of a bar magnet points away from the north pole and toward the south pole.

EXECUTE: Between the poles of the magnet, the magnetic field points to the right. Using the fingertips of your right hand, rotate the current vector by 90° into the direction of the magnetic field vector. Your thumb points downward—which is the direction of the magnetic force.

EVALUATE If the two magnets had their poles interchanged, then the force would be upward.

27.39. **IDENTIFY and SET UP:** The magnetic force is given by Eq.(27.19). $F_I = mg$ when the bar is just ready to levitate. When I becomes larger, $F_I > mg$ and $F_I - mg$ is the net force that accelerates the bar upward. Use Newton's 2nd law to find the acceleration.

(a) EXECUTE: $IlB = mg,\ I = \dfrac{mg}{lB} = \dfrac{(0.750\text{ kg})(9.80\text{ m/s}^2)}{(0.500\text{ m})(0.450\text{ T})} = 32.67\text{ A}$

$\mathcal{E} = IR = (32.67\text{ A})(25.0\ \Omega) = 817\text{ V}$

(b) $R = 2.0\ \Omega,\ I = \mathcal{E}/R = (816.7\text{ V})/(2.0\ \Omega) = 408\text{ A}$

$F_I = IlB = 92\text{ N}$

$a = (F_I - mg)/m = 113\text{ m/s}^2$

EVALUATE: I increases by over an order of magnitude when R changes to $F_I >> mg$ and a is an order of magnitude larger than g.

27.40. **IDENTIFY:** The magnetic force $\vec{F}_B$ must be upward and equal to mg. The direction of $\vec{F}_B$ is determined by the direction of I in the circuit.

SET UP: $F_B = IlB\sin\phi$, with $\phi = 90°$. $I = \dfrac{V}{R}$, where V is the battery voltage.

EXECUTE: **(a)** The forces are shown in Figure 27.40. The current I in the bar must be to the right to produce $\vec{F}_B$ upward. To produce current in this direction, point a must be the positive terminal of the battery.

(b) $F_B = mg$. $IlB = mg$. $m = \dfrac{IlB}{g} = \dfrac{VlB}{Rg} = \dfrac{(175\text{ V})(0.600\text{ m})(1.50\text{ T})}{(5.00\ \Omega)(9.80\text{ m/s}^2)} = 3.21\text{ kg}$.

EVALUATE: If the battery had opposite polarity, with point a as the negative terminal, then the current would be clockwise and the magnetic force would be downward.

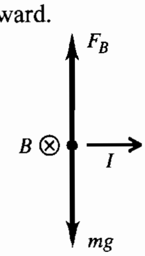


Figure 27.40

27.41. **IDENTIFY:** Apply $\vec{F} = I\vec{l} \times \vec{B}$ to each segment of the conductor: the straight section parallel to the x axis, the semicircular section and the straight section that is perpendicular to the plane of the figure in Example 27.8.

SET UP: $\vec{B} = B_x\hat{i}$. The force is zero when the current is along the direction of $\vec{B}$.

EXECUTE: **(a)** The force on the straight section along the $-x$-axis is zero. For the half of the semicircle at negative x the force is out of the page. For the half of the semicircle at positive x the force is into the page. The net force on the semicircular section is zero. The force on the straight section that is perpendicular to the plane of the figure is in the $-y$-direction and has magnitude $F = ILB$. The total magnetic force on the conductor is ILB, in the $-y$-direction.

EVALUATE: **(b)** If the semicircular section is replaced by a straight section along the x -axis, then the magnetic force on that straight section would be zero, the same as it is for the semicircle.

27.42. **IDENTIFY:** $\tau = IAB\sin\phi$. The magnetic moment of the loop is $\mu = IA$.

SET UP: Since the plane of the loop is parallel to the field, the field is perpendicular to the normal to the loop and $\phi = 90°$.

EXECUTE: **(a)** $\tau = IAB = (6.2\text{ A})(0.050\text{ m})(0.080\text{ m})(0.19\text{ T}) = 4.7\times10^{-3}\text{ N}\cdot\text{m}$

(b) $\mu = IA = (6.2\text{ A})(0.050\text{ m})(0.080\text{ m}) = 0.025\text{ A}\cdot\text{m}^2$

(c) Maximum area is when the loop is circular. $R = \dfrac{0.050\text{ m} + 0.080\text{ m}}{\pi} = 0.0414\text{ m}$

$A = \pi R^2 = 5.38\times10^{-3}\text{ m}^2$ and $T = (6.2\text{ A})(5.38\times10^{-3}\text{ m}^2)(0.19\text{ T}) = 6.34\times10^{-3}\text{ N}\cdot\text{m}$

EVALUATE: The torque is a maximum when the field is in the plane of the loop and $\phi = 90°$.

27.43. **IDENTIFY:** The period is $T = 2\pi r/v$, the current is Q/t and the magnetic moment is $\mu = IA$

SET UP: The electron has charge $-e$. The area enclosed by the orbit is πr^2.

EXECUTE: **(a)** $T = 2\pi r/v = 1.5\times10^{-16}\text{ s}$

(b) Charge $-e$ passes a point on the orbit once during each period, so $I = Q/t = e/t = 1.1\text{ mA}$.

(c) $\mu = IA = I\pi r^2 = 9.3\times10^{-24}\text{ A}\cdot\text{m}^2$

EVALUATE: Since the electron has negative charge, the direction of the current is opposite to the direction of motion of the electron.

27.44. **IDENTIFY:** $\tau = IAB\sin\phi$, where ϕ is the angle between $\vec{B}$ and the normal to the loop.

SET UP: The coil as viewed along the axis of rotation is shown in Figure 27.44a for its original position and in Figure 27.44b after it has rotated 30.0°.

EXECUTE: **(a)** The forces on each side of the coil are shown in Figure 27.44a. $\vec{F}_1 + \vec{F}_2 = 0$ and $\vec{F}_3 + \vec{F}_4 = 0$. The net force on the coil is zero. $\phi = 0^\circ$ and $\sin\phi = 0$, so $\tau = 0$. The forces on the coil produce no torque.

(b) The net force is still zero. $\phi = 30.0^\circ$ and the net torque is

$\tau = (1)(1.40\text{ A})(0.220\text{ m})(0.350\text{ m})(1.50\text{ T})\sin 30.0^\circ = 0.0808\text{ N}\cdot\text{m}$. The net torque is clockwise in Figure 27.44b and is directed so as to increase the angle ϕ.

EVALUATE: For any current loop in a uniform magnetic field the net force on the loop is zero. The torque on the loop depends on the orientation of the plane of the loop relative to the magnetic field direction.

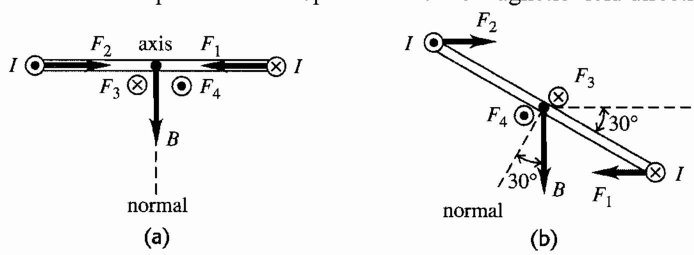


Figure 27.44

27.45. **IDENTIFY:** The magnetic field exerts a torque on the current-carrying coil, which causes it to turn. We can use the rotational form of Newton's second law to find the angular acceleration of the coil.

SET UP: The magnetic torque is given by $\vec{\tau} = \vec{\mu} \times \vec{B}$, and the rotational form of Newton's second law is $\sum \tau = I\alpha$. The magnetic field is parallel to the plane of the loop.

EXECUTE: **(a)** The coil rotates about axis A_2 because the only torque is along top and bottom sides of the coil.
(b) To find the moment of inertia of the coil, treat the two 1.00-m segments as point-masses (since all the points in them are 0.250 m from the rotation axis) and the two 0.500-m segments as thin uniform bars rotated about their centers. Since the coil is uniform, the mass of each segment is proportional to its fraction of the total perimeter of the coil. Each 1.00-m segment is 1/3 of the total perimeter, so its mass is (1/3)(210 g) = 70 g = 0.070 kg. The mass of each 0.500-m segment is half this amount, or 0.035 kg. The result is

$$I = 2(0.070\text{ kg})(0.250\text{ m})^2 + 2\tfrac{1}{12}(0.035\text{ kg})(0.500\text{ m})^2 = 0.0102\text{ kg}\cdot\text{m}^2$$

The torque is

$$|\vec{\tau}| = |\vec{\mu} \times \vec{B}| = IAB\sin 90^\circ = (2.00\text{ A})(0.500\text{ m})(1.00\text{ m})(3.00\text{ T}) = 3.00\text{ N}\cdot\text{m}$$

Using the above values, the rotational form of Newton's second law gives

$$\alpha = \frac{\tau}{I} = 290\text{ rad/s}^2$$

EVALUATE: This angular acceleration will not continue because the torque changes as the coil turns.

27.46. **IDENTIFY:** $\vec{\tau} = \vec{\mu} \times \vec{B}$ and $U = -\mu B\cos\phi$, where $\mu = NIB$. $\tau = \mu B\sin\phi$.

SET UP: ϕ is the angle between $\vec{B}$ and the normal to the plane of the loop.

EXECUTE: **(a)** $\phi = 90^\circ$. $\tau = NIAB\sin(90^\circ) = NIAB$, direction $\hat{k} \times \hat{j} = -\hat{i}$. $U = -\mu B\cos\phi = 0$.

(b) $\phi = 0$. $\tau = NIAB\sin(0) = 0$, no direction. $U = -\mu B\cos\phi = -NIAB$.

(c) $\phi = 90^\circ$. $\tau = NIAB\sin(90^\circ) = NIAB$, direction $-\hat{k} \times \hat{j} = \hat{i}$. $U = -\mu B\cos\phi = 0$.

(d) $\phi = 180^\circ$: $\tau = NIAB\sin(180^\circ) = 0$, no direction, $U = -\mu B\cos(180^\circ) = NIAB$.

EVALUATE: When τ is maximum, $U = 0$. When $|U|$ is maximum, $\tau = 0$.

27.47. **IDENTIFY and SET UP:** The potential energy is given by Eq.(27.27): $U = \vec{\mu} \cdot \vec{B}$. The scalar product depends on the angle between $\vec{\mu}$ and $\vec{B}$.

EXECUTE: For $\vec{\mu}$ and $\vec{B}$ parallel, $\phi = 0^\circ$ and $\vec{\mu} \cdot \vec{B} = \mu B\cos\phi = \mu B$. For $\vec{\mu}$ and $\vec{B}$ antiparallel, $\phi = 180^\circ$ and $\vec{\mu} \cdot \vec{B} = \mu B\cos\phi = -\mu B$.

$U_1 = +\mu B$, $U_2 = -\mu B$

$\Delta U = U_2 - U_1 = -2\mu B = -2(1.45\text{ A}\cdot\text{m}^2)(0.835\text{ T}) = -2.42\text{ J}$

EVALUATE: U is maximum when $\vec{\mu}$ and $\vec{B}$ are antiparallel and minimum when they are parallel. When the coil is rotated as specified its magnetic potential energy decreases.

27.48. **IDENTIFY:** Apply Eq.(27.29) in order to calculate I. The power drawn from the line is $P_{\text{supplied}} = IV_{ab}$. The mechanical power is the power supplied minus the I^2r electrical power loss in the internal resistance of the motor.

SET UP: $V_{ab} = 120\text{V}$, $\mathcal{E} = 105\text{ V}$, and $r = 3.2\ \Omega$.

EXECUTE: **(a)** $V_{ab} = \mathcal{E} + Ir \Rightarrow I = \dfrac{V_{ab} - \mathcal{E}}{r} = \dfrac{120\text{ V} - 105\text{ V}}{3.2\ \Omega} = 4.7\text{ A}.$

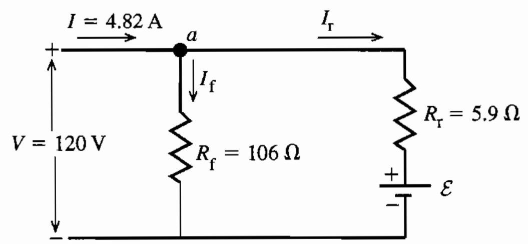
(b) $P_{\text{supplied}} = IV_{ab} = (4.7\text{ A})(120\text{ V}) = 564\text{ W}.$

(c) $P_{\text{mech}} = IV_{ab} - I^2r = 564\text{ W} - (4.7\text{ A})^2(3.2\ \Omega) = 493\text{ W}.$

EVALUATE: If the rotor isn't turning, when the motor is first turned on or if the rotor bearings fail, then $\mathcal{E} = 0$ and $I = \dfrac{120\text{V}}{3.2\ \Omega} = 37.5\text{ A}$. This large current causes large I^2r heating and can trip the circuit breaker.

27.49. **IDENTIFY:** The circuit consists of two parallel branches with the potential difference of 120 V applied across each. One branch is the rotor, represented by a resistance R_{r} and an induced emf that opposes the applied potential. Apply the loop rule to each parallel branch and use the junction rule to relate the currents through the field coil and through the rotor to the 4.82 A supplied to the motor.

SET UP: The circuit is sketched in Figure 27.49.



$\mathcal{E}$ is the induced emf developed by the motor. It is directed so as to oppose the current through the rotor.

Figure 27.49

EXECUTE: **(a)** The field coils and the rotor are in parallel with the applied potential difference V, so $V = I_{\text{f}}R_{\text{f}}$.

$I_{\text{f}} = \dfrac{V}{R_{\text{f}}} = \dfrac{120\text{ V}}{106\ \Omega} = 1.13\text{ A}.$

(b) Applying the junction rule to point a in the circuit diagram gives $I - I_{\text{f}} - I_{\text{r}} = 0.$

$I_{\text{r}} = I - I_{\text{f}} = 4.82\text{ A} - 1.13\text{ A} = 3.69\text{ A}.$

(c) The potential drop across the rotor, $I_{\text{r}}R_{\text{r}} + \mathcal{E}$, must equal the applied potential difference $V: V = I_{\text{r}}R_{\text{r}} + \mathcal{E}$

$\mathcal{E} = V - I_{\text{r}}R_{\text{r}} = 120\text{ V} - (3.69\text{ A})(5.9\ \Omega) = 98.2\text{ V}$

(d) The mechanical power output is the electrical power input minus the rate of dissipation of electrical energy in the resistance of the motor:

electrical power input to the motor

$P_{\text{in}} = IV = (4.82\text{ A})(120\text{ V}) = 578\text{ W}$

electrical power loss in the two resistances

$P_{\text{loss}} = I_{\text{f}}^2R_{\text{f}} + I_{\text{f}}^2R = (1.13\text{ A})^2(106\ \Omega) + (3.69\text{ A})^2(5.9\ \Omega) = 216\text{ W}$

mechanical power output

$P_{\text{out}} = P_{\text{in}} - P_{\text{loss}} = 578\text{ W} - 216\text{ W} = 362\text{ W}$

The mechanical power output is the power associated with the induced emf $\mathcal{E}$

$P_{\text{out}} = P_{\mathcal{E}} = \mathcal{E}I_{\text{r}} = (98.2\text{ V})(3.69\text{ A}) = 362\text{ W}$, which agrees with the above calculation.

EVALUATE: The induced emf reduces the amount of current that flows through the rotor. This motor differs from the one described in Example 27.12. In that example the rotor and field coils are connected in series and in this problem they are in parallel.

27.50. **IDENTIFY:** The field and rotor coils are in parallel, so $V_{ab} = I_{\text{f}}R_{\text{f}} = \mathcal{E} + I_{\text{r}}R_{\text{r}}$ and $I = I_{\text{f}} + I_{\text{r}}$, where I is the current drawn from the line. The power input to the motor is $P = V_{ab}I$. The power output of the motor is the power input minus the electrical power losses in the resistances and friction losses.

SET UP: $V_{ab} = 120\text{ V}$. $I = 4.82\text{ A}$.

EXECUTE: **(a)** Field current $I_{\text{f}} = \dfrac{120\text{ V}}{218\ \Omega} = 0.550\text{ A}.$

(b) Rotor current $I_r = I_{total} - I_f = 4.82\text{ A} - 0.550\text{ A} = 4.27\text{ A}$.

(c) $V = \mathcal{E} + I_r R_r$ and $\mathcal{E} = V - I_r R_r = 120\text{ V} - (4.27\text{ A})(5.9\ \Omega) = 94.8\text{ V}$.

(d) $P_f = I_f^2 R_f = (0.550\text{ A})^2(218\ \Omega) = 65.9\text{ W}$.

(e) $P_r = I_r^2 R_r = (4.27\text{ A})^2(5.9\ \Omega) = 108\text{ W}$.

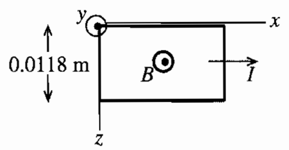
(f) Power input = (120 V) (4.82 A) = 578 W.

(g) Efficiency $= \dfrac{P_{output}}{P_{input}} = \dfrac{(578\text{ W} - 65.9\text{ W} - 108\text{ W} - 45\text{ W})}{578\text{ W}} = \dfrac{359\text{ W}}{578\text{ W}} = 0.621$.

EVALUATE: I^2R losses in the resistance of the rotor and field coils are larger than the friction losses for this motor.

27.51. **IDENTIFY:** The drift velocity is related to the current density by Eq.(25.4). The electric field is determined by the requirement that the electric and magnetic forces on the current-carrying charges are equal in magnitude and opposite in direction.

(a) SET UP: The section of the silver ribbon is sketched in Figure 27.51a.



$J_x = n|q|v_d$

so $v_d = \dfrac{J_x}{n|q|}$

Figure 27.51a

EXECUTE: $J_x = \dfrac{I}{A} = \dfrac{I}{y_1 z_1} = \dfrac{120\text{ A}}{(0.23\times10^{-3}\text{ m})(0.0118\text{ m})} = 4.42\times10^7\text{ A/m}^2$

$$v_d = \frac{J_x}{n|q|} = \frac{4.42\times10^7\text{ A/m}^2}{(5.85\times10^{28}/\text{m}^3)(1.602\times10^{-19}\text{ C})} = 4.7\times10^{-3}\text{ m/s} = 4.7\text{ mm/s}$$

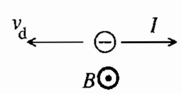
(b) magnitude of $\vec{E}$

$|q|E_z = |q|v_d B_y$

$E_z = v_d B_y = (4.7\times10^{-3}\text{ m/s})(0.95\text{ T}) = 4.5\times10^{-3}\text{ V/m}$

direction of $\vec{E}$

The drift velocity of the electrons is in the opposite direction to the current, as shown in Figure 27.51b.

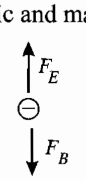


$\vec{v}\times\vec{B}\uparrow$

$\vec{F}_B = q\vec{v}\times\vec{B} = -e\vec{v}\times\vec{B}\downarrow$

Figure 27.51b

The directions of the electric and magnetic forces on an electron in the ribbon are shown in Figure 27.51c.



$\vec{F}_E$ must oppose $\vec{F}_B$ so $\vec{F}_E$ is in the $-z$-direction

Figure 27.51c

$\vec{F}_E = q\vec{E} = -e\vec{E}$ so $\vec{E}$ is opposite to the direction of $\vec{F}_E$ and thus $\vec{E}$ is in the $+z$-direction.

(c) The Hall emf is the potential difference between the two edges of the strip (at $z = 0$ and $z = z_1$) that results from the electric field calculated in part (b). $\mathcal{E}_{Hall} = Ez_1 = (4.5\times10^{-3}\text{ V/m})(0.0118\text{ m}) = 53\ \mu\text{V}$

EVALUATE: Even though the current is quite large the Hall emf is very small. Our calculated Hall emf is more than an order of magnitude larger than in Example 27.13. In this problem the magnetic field and current density are larger than in the example, and this leads to a larger Hall emf.

27.52. **IDENTIFY:** Apply Eq.(27.30).

SET UP: $A = y_1 z_1$. $E = \mathcal{E}/z_1$. $|q| = e$.

EXECUTE: $n = \dfrac{J_x B_y}{|q|E_z} = \dfrac{IB_y}{A|q|E_z} = \dfrac{IB_y z_1}{A|q|\mathcal{E}} = \dfrac{IB_y}{y_1|q|\mathcal{E}}$

$$n = \frac{(78.0\text{ A})(2.29\text{ T})}{(2.3\times10^{-4}\text{ m})(1.6\times10^{-19}\text{ C})(1.31\times10^{-4}\text{ V})} = 3.7\times10^{28}\text{ electrons/m}^3$$

EVALUATE: The value of n for this metal is about one-third the value of n calculated in Example 27.12 for copper.

27.53. **(a) IDENTIFY:** Use Eq.(27.2) to relate $\vec{v}$, $\vec{B}$, and $\vec{F}$.

SET UP: The directions of $\vec{v}_1$ and $\vec{F}_1$ are shown in Figure 27.53a.

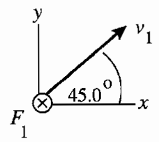


Figure 27.53a

$\vec{F} = q\vec{v}\times\vec{B}$ says that $\vec{F}$ is perpendicular to $\vec{v}$ and $\vec{B}$. The information given here means that $\vec{B}$ can have no z-component.

The directions of $\vec{v}_2$ and $\vec{F}_2$ are shown in Figure 27.53b.

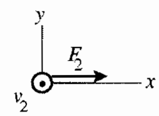


Figure 27.53b

$\vec{F}$ is perpendicular to $\vec{v}$ and $\vec{B}$, so $\vec{B}$ can have no x-component.

Both pieces of information taken together say that $\vec{B}$ is in the y-direction; $\vec{B} = B_y\hat{j}$.

EXECUTE: Use the information given about $\vec{F}_2$ to calculate F_y: $\vec{F}_2 = F_2\hat{i}$, $\vec{v}_2 = v_2\hat{k}$, $\vec{B} = B_y\hat{j}$.

$\vec{F}_2 = q\vec{v}_2\times\vec{B}$ says $F_2\hat{i} = qv_2B_y\hat{k}\times\hat{j} = qv_2B_y(-\hat{i})$ and $F_2 = -qv_2B_y$

$B_y = -F_2/(qv_2) = -F_2/(qv_1)$. $\vec{B}$ has the maginitude $F_2/(qv_1)$ and is in the $-y$-direction.

(b) $F_1 = qvB\sin\phi = qv_1|B_y|/\sqrt{2} = F_2/\sqrt{2}$

EVALUATE: $v_1 = v_2$. $\vec{v}_2$ is perpendicular to $\vec{B}$ whereas only the component of $\vec{v}_1$ perpendicular to $\vec{B}$ contributes to the force, so it is expected that $F_2 > F_1$, as we found.

27.54. **IDENTIFY:** Apply $\vec{F} = q\vec{v}\times\vec{B}$.

SET UP: $B_x = 0.450\text{ T}$, $B_y = 0$ and $B_z = 0$.

EXECUTE: $F_x = q(v_yB_z - v_zB_y) = 0$.

$F_y = q(v_zB_x - v_xB_z) = (9.45\times10^{-8}\text{ C})(5.85\times10^{4}\text{ m/s})(0.450\text{ T}) = 2.49\times10^{-3}\text{ N}$.

$F_z = q(v_xB_y - v_yB_x) = -(9.45\times10^{-8}\text{ C})(-3.11\times10^{4}\text{ m/s})(0.450\text{ T}) = 1.32\times10^{-3}\text{ N}$.

EVALUATE: $\vec{F}$ is perpendicular to both $\vec{v}$ and $\vec{B}$. We can verify that $\vec{F}\cdot\vec{v} = 0$. Since $\vec{B}$ is along the x-axis, v_x does not affect the force components.

27.55. **IDENTIFY:** The sum of the magnetic, electrical, and gravitational forces must be zero to aim at and hit the target.
SET UP: The magnetic field must point to the left when viewed in the direction of the target for no net force. The net force is zero, so $\sum F = F_B - F_E - mg = 0$ and $qvB - qE - mg = 0$.

EXECUTE: Solving for B gives

$$B = \frac{qE + mg}{qv} = \frac{(2500\times10^{-6}\text{ C})(27.5\text{ N/C}) + (0.0050\text{ kg})(9.80\text{ m/s}^2)}{(2500\times10^{-6}\text{ C})(12.8\text{ m/s})} = 3.7\text{ T}$$

The direction should be perpendicular to the initial velocity of the coin.
EVALUATE: This is a very strong magnetic field, but achievable in some labs.

27.56. **IDENTIFY:** Apply $R = mv/|q|B$. $\omega = v/R$

SET UP: $1\text{ eV} = 1.60\times10^{-19}\text{ J}$

EXECUTE: **(a)** $K = 2.7\text{ MeV} = (2.7\times10^{6}\text{eV})(1.6\times10^{-19}\text{ J/eV}) = 4.32\times10^{-13}\text{ J}$.

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(4.32\times10^{-13}\text{ J})}{1.67\times10^{-27}\text{ kg}}} = 2.27\times10^{7}\text{ m/s}.$$

$$R = \frac{mv}{qB} = \frac{(1.67\times10^{-27}\text{ kg})(2.27\times10^{7}\text{ m/s})}{(1.6\times10^{-19}\text{ C})(3.5\text{ T})} = 0.068\text{ m. Also, } \omega = \frac{v}{R} = \frac{2.27\times10^{7}\text{ m/s}}{0.068\text{ m}} = 3.34\times10^{8}\text{ rad/s.}$$

(b) If the energy reaches the final value of 5.4 MeV, the velocity increases by $\sqrt{2}$, as does the radius, to 0.096 m. The angular frequency is unchanged from part (a) so is 3.34×10^{8} rad/s.

EVALUATE: $\omega = |q|B/m$, so ω is independent of the energy of the protons. The orbit radius increases when the energy of the proton increases.

27.57. **(a) IDENTIFY and SET UP:** The maximum radius of the orbit determines the maximum speed v of the protons. Use Newton's 2nd law and $a_c = v^2/R$ for circular motion to relate the variables. The energy of the particle is the kinetic energy $K = \frac{1}{2}mv^2$.

EXECUTE: $\sum \vec{F} = m\vec{a}$ gives $|q|vB = m(v^2/R)$

$v = \dfrac{|q|BR}{m} = \dfrac{(1.60\times10^{-19}\text{ C})(0.85\text{ T})(0.40\text{ m})}{1.67\times10^{-27}\text{ kg}} = 3.257\times10^7$ m/s. The kinetic energy of a proton moving with this speed is $K = \frac{1}{2}mv^2 = \frac{1}{2}(1.67\times10^{-27}\text{ kg})(3.257\times10^7\text{ m/s})^2 = 8.9\times10^{-13}\text{ J} = 5.6\text{ MeV}$

(b) The time for one revolution is the period $T = \dfrac{2\pi R}{v} = \dfrac{2\pi(0.40\text{ m})}{3.257\times10^7\text{ m/s}} = 7.7\times10^{-8}\text{ s}$

(c) $K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\dfrac{|q|BR}{m}\right)^2 = \frac{1}{2}\dfrac{|q|^2B^2R^2}{m}$. Or, $B = \dfrac{\sqrt{2Km}}{|q|R}$. B is proportional to $\sqrt{K}$, so if K is increased by a factor of 2 then B must be increased by a factor of $\sqrt{2}$. $B = \sqrt{2}(0.85\text{ T}) = 1.2\text{ T}$.

(d) $v = \dfrac{|q|BR}{m} = \dfrac{(3.20\times10^{-19}\text{ C})(0.85\text{ T})(0.40\text{ m})}{6.65\times10^{-27}\text{ kg}} = 1.636\times10^7\text{ m/s}$

$K = \frac{1}{2}mv^2 = \frac{1}{2}(6.65\times10^{-27}\text{ kg})(1.636\times10^7\text{ m/s})^2 = 8.9\times10^{-13}\text{ J} = 5.5\text{ MeV}$, the same as the maximum energy for protons.

EVALUATE: We can see that the maximum energy must be approximately the same as follows: From part (c), $K = \frac{1}{2}m\left(\dfrac{|q|BR}{m}\right)^2$. For alpha particles $|q|$ is larger by a factor of 2 and m is larger by a factor of 4 (approximately). Thus $|q|^2/m$ is unchanged and K is the same.

27.58. **IDENTIFY:** Apply $\vec{F} = q\vec{v}\times\vec{B}$.

SET UP: $\vec{v} = -v\hat{j}$

EXECUTE: **(a)** $\vec{F} = -qv[B_x(\hat{j}\times\hat{i}) + B_y(\hat{j}\times\hat{j}) + B_z(\hat{j}\times\hat{k})] = qvB_x\hat{k} - qvB_z\hat{i}$

(b) $B_x > 0, B_z < 0$, sign of B_y doesn't matter.

(c) $\vec{F} = |q|vB_x\hat{i} - |q|vB_x\hat{k}$ and $|\vec{F}| = \sqrt{2}|q|vB_x$.

EVALUATE: $\vec{F}$ is perpendicular to $\vec{v}$, so $\vec{F}$ has no y-component.

27.59. **IDENTIFY:** The contact at a will break if the bar rotates about b. The magnetic field is directed out of the page, so the magnetic torque is counterclockwise, whereas the gravity torque is clockwise in the figure in the problem. The maximum current corresponds to zero net torque, in which case the torque due to gravity is just equal to the torque due to the magnetic field.

SET UP: The magnetic force is perpendicular to the bar and has moment arm $l/2$, where $l = 0.750\text{ m}$ is the length of the bar. The gravity torque is $mg\left(\dfrac{l}{2}\cos 60.0°\right)$

EXECUTE: $\tau_{\text{gravity}} = \tau_B$ and $mg\dfrac{l}{2}\cos 60.0° = IlB\sin 90°\dfrac{l}{2}$. This gives

$I = \dfrac{mg\cos 60.0°}{lB\sin 90°} = \dfrac{(0.458\text{kg})(9.80\text{ m/s}^2)(\cos 60.0°)}{(0.750\text{ m})(1.55\text{ T})(1)} = 1.93\text{ A}$

EVALUATE: Once contact is broken, the magnetic torque ceases. The $90.0°$ angle in the expression for τ_B is the angle between the direction of I and the direction of $\vec{B}$.

27.60. **IDENTIFY:** Apply $R = \dfrac{mv}{|q|B}$.

SET UP: Assume $D << R$

EXECUTE: **(a)** The path is sketched in Figure 27.60.

(b) Motion is circular: $x^2+y^2=R^2 \Rightarrow x=D \Rightarrow y_1=\sqrt{R^2-D^2}$ (path of deflected particle)

$y_2=R$ (equation for tangent to the circle, path of undeflected particle).

$d=y_2-y_1=R-\sqrt{R^2-D^2}=R-R\sqrt{1-\frac{D^2}{R^2}}=R\left[1-\sqrt{1-\frac{D^2}{R^2}}\right]$. If $R>>D$, $d\approx R\left[1-\left(1-\frac{1}{2}\frac{D^2}{R^2}\right)\right]=\frac{D^2}{2R}$. For a particle moving in a magnetic field, $R=\frac{mv}{qB}$. But $\frac{1}{2}mv^2=qV$, so $R=\frac{1}{B}\sqrt{\frac{2mV}{q}}$. Thus, the deflection

$d\approx\frac{D^2B}{2}\sqrt{\frac{q}{2mV}}=\frac{D^2B}{2}\sqrt{\frac{e}{2mV}}$.

(c) $d=\frac{(0.50\text{ m})^2(5.0\times10^{-5}\text{ T})}{2}\sqrt{\frac{(1.6\times10^{-19}\text{ C})}{2(9.11\times10^{-31}\text{ kg})(750\text{ V})}}=0.067\text{ m}=6.7\text{ cm}$. $d\approx 13\%$ of D, which is fairly significant.

EVALUATE: In part (c), $R=\frac{1}{B}\sqrt{\frac{2mV}{e}}=\frac{D^2}{2d}=\left(\frac{D}{2d}\right)D=3.7D$ and $\left(\frac{R}{D}\right)^2=14$, so the approximation made in part (b) is valid.

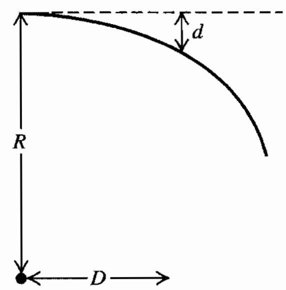


Figure 27.60

27.61. **IDENTIFY** and **SET UP:** Use Eq.(27.2) to relate $q,\vec{v},\vec{B}$ and $\vec{F}$. The force $\vec{F}$ and $\vec{a}$ are related by Newton's 2nd law.

$\vec{B}=-(0.120\text{ T})\hat{k}, \vec{v}=(1.05\times10^6\text{ m/s})(-3\hat{i}+4\hat{j}+12\hat{k})$, $F=1.25$ N

(a) EXECUTE: $\vec{F}=q\vec{v}\times\vec{B}$

$\vec{F}=q(-0.120\text{ T})(1.05\times10^6\text{ m/s})(-3\hat{i}\times\hat{k}+4\hat{j}\times\hat{k}+12\hat{k}\times\hat{k})$

$\hat{i}\times\hat{k}=-\hat{j}, \hat{j}\times\hat{k}=\hat{i}, \hat{k}\times\hat{k}=0$

$\vec{F}=-q(1.26\times10^5\text{ N/C})(+3\hat{j}+4\hat{i})=-q(1.26\times10^5\text{ N/C})(+4\hat{i}+3\hat{j})$

The magnitude of the vector $+4\hat{i}+3\hat{j}$ is $\sqrt{3^2+4^2}=5$. Thus $F=-q(1.26\times10^5\text{ N/C})(5)$.

$$q=-\frac{F}{5(1.26\times10^5\text{ N/C})}=-\frac{1.25\text{ N}}{5(1.26\times10^5\text{ N/C})}=-1.98\times10^{-6}\text{ C}$$

(b) $\sum\vec{F}=m\vec{a}$ so $\vec{a}=\vec{F}/m$

$\vec{F}=-q(1.26\times10^5\text{ N/C})(+4\hat{i}+3\hat{j})=-(-1.98\times10^{-6}\text{ C})(1.26\times10^5\text{ N/C})(+4\hat{i}+3\hat{j})=+0.250\text{ N}(+4\hat{i}+3\hat{j})$

Then $\vec{a}=\vec{F}/m=\left(\frac{0.250\text{ N}}{2.58\times10^{-15}\text{ kg}}\right)(+4\hat{i}+3\hat{j})=(9.69\times10^{13}\text{ m/s}^2)(+4\hat{i}+3\hat{j})$

(c) IDENTIFY and **SET UP:** $\vec{F}$ is in the *xy*-plane, so in the *z*-direction the particle moves with constant speed 12.6×10^6 m/s. In the *xy*-plane the force $\vec{F}$ causes the particle to move in a circle, with $\vec{F}$ directed in towards the center of the circle.

EXECUTE: $\sum\vec{F}=m\vec{a}$ gives $F=m(v^2/R)$ and $R=mv^2/F$

$v^2=v_x^2+v_y^2=(-3.15\times10^6\text{ m/s})^2+(+4.20\times10^6\text{ m/s})^2=2.756\times10^{13}\text{ m}^2/\text{s}^2$

$F=\sqrt{F_x^2+F_y^2}=(0.250\text{ N})\sqrt{4^2+3^2}=1.25\text{ N}$

$$R=\frac{mv^2}{F}=\frac{(2.58\times10^{-15}\text{ kg})(2.756\times10^{13}\text{ m}^2/\text{s}^2)}{1.25\text{ N}}=0.0569\text{ m}=5.69\text{ cm}$$

(d) IDENTIFY and SET UP: By Eq.(27.12) the cyclotron frequency is $f = \omega/2\pi = v/2\pi R$.

EXECUTE: The circular motion is in the xy-plane, so $v = \sqrt{v_x^2 + v_y^2} = 5.25\times10^6$ m/s.

$$f = \frac{v}{2\pi R} = \frac{5.25\times10^6 \text{ m/s}}{2\pi(0.0569 \text{ m})} = 1.47\times10^7 \text{ Hz, and } \omega = 2\pi f = 9.23\times10^7 \text{ rad/s}$$

(e) IDENTIFY and SET UP Compare t to the period T of the circular motion in the xy-plane to find the x and y coordinates at this t. In the z-direction the particle moves with constant speed, so $z = z_0 + v_z t$.

EXECUTE: The period of the motion in the xy-plane is given by $T = \frac{1}{f} = \frac{1}{1.47\times10^7 \text{ Hz}} = 6.80\times10^{-8}$ s

In $t = 2T$ the particle has returned to the same x and y coordinates. The z-component of the motion is motion with a constant velocity of $v_z = +12.6\times10^6$ m/s. Thus $z = z_0 + v_z t = 0 + (12.6\times10^6 \text{ m/s})(2)(6.80\times10^{-8} \text{ s}) = +1.71$ m.

The coordinates at $t = 2T$ are $x = R, y = 0, z = +1.71$ m.

EVALUATE: The circular motion is in the plane perpendicular to $\vec{B}$. The radius of this motion gets smaller when B increases and it gets larger when v increases. There is no magnetic force in the direction of $\vec{B}$ so the particle moves with constant velocity in that direction. The superposition of circular motion in the xy-plane and constant speed motion in the z-direction is a helical path.

27.62. **IDENTIFY:** The net magnetic force on the wire is the vector sum of the force on the straight segment plus the force on the curved section. We must integrate to get the force on the curved section.

SET UP: $\sum F = F_{\text{straight, top}} + F_{\text{curved}} + F_{\text{straight, bottom}}$ and $F_{\text{straight, top}} = F_{\text{straight, bottom}} = iL_{\text{straight}}B$. $F_{\text{curved}, x} = \int_0^\pi iRB\sin\theta d\theta = 2iRB$

(the same as if it were a straight segment $2R$ long) and $F_y = 0$ due to symmetry. Therefore, $F = 2IL_{\text{straight}}B + 2IRB$

EXECUTE: Using $L_{\text{straight}} = 0.55$ m, $R = 0.95$ m, $I = 3.40$ A, and $B = 2.20$ T gives $F = 22$ N, to right.

EVALUATE: Notice that the curve has no effect on the force. In other words, the force is the same as if the wire were simply a straight wire 3.00 m long.

27.63. **IDENTIFY:** $\tau = NIAB\sin\phi$.

SET UP: The area A is related to the diameter D by $A = \frac{1}{4}\pi D^2$.

EXECUTE: $\tau = NI(\frac{1}{4}\pi D^2)B\sin\phi$. τ is proportional to D^2. Increasing D by a factor of 3 increases τ by a factor of $3^2 = 9$.

EVALUATE: The larger diameter means larger length of wire in the loop and also larger moment arms because parts of the loop are farther from the axis.

27.64. **IDENTIFY:** Apply $\vec{F} = q\vec{v}\times\vec{B}$

SET UP: $\vec{v} = v\hat{k}$

EXECUTE: **(a)** $\vec{F} = -qvB_y\hat{i} + qvB_x\hat{j}$. But $\vec{F} = 3F_0\hat{i} + 4F_0\hat{j}$, so $3F_0 = -qvB_y$ and $4F_0 = qvB_x$

Therefore, $B_y = -\frac{3F_0}{qv}$, $B_x = \frac{4F_0}{qv}$ and B_z is undetermined.

(b) $B = \frac{6F_0}{qv} = \sqrt{B_x^2 + B_y^2 + B_z^2} = \frac{F_0}{qv}\sqrt{9 + 16 + \left(\frac{qv}{F_0}\right)^2 B_z^2} = \frac{F_0}{qv}\sqrt{25 + \left(\frac{qv}{F_0}\right)^2 B_z^2}$, so $B_z = \pm\frac{11F_0}{qv}$.

EVALUATE: The force doesn't depend on B_z, since $\vec{v}$ is along the z-direction.

27.65. **IDENTIFY:** For the velocity selector, $E = vB$. For the circular motion in the field B', $R = \frac{mv}{|q|B'}$.

SET UP: $B = B' = 0.701$ T.

EXECUTE: $v = \frac{E}{B} = \frac{1.88\times10^4 \text{ N/C}}{0.701 \text{ T}} = 2.68\times10^4$ m/s. $R = \frac{mv}{qB'}$, so

$$R_{82} = \frac{82(1.66\times10^{-27} \text{ kg})(2.68\times10^4 \text{ m/s})}{(1.60\times10^{-19} \text{ C})(0.701 \text{ T})} = 0.0325 \text{ m}.$$

$$R_{84} = \frac{84(1.66\times10^{-27} \text{ kg})(2.68\times10^4 \text{ m/s})}{(1.60\times10^{-19} \text{ C})(0.701 \text{ T})} = 0.0333 \text{ m}.$$

$$R_{86} = \frac{86(1.66\times10^{-27} \text{ kg})(2.68\times10^4 \text{ m/s})}{(1.60\times10^{-19} \text{ C})(0.701 \text{ T})} = 0.0341 \text{ m}.$$

The distance between two adjacent lines is $2\Delta R = 1.6$ mm.

EVALUATE: The distance between the ^{82}Kr line and the ^{84}Kr line is 1.6 mm and the distance between the ^{84}Kr line and the ^{86}Kr line is 1.6 mm. Adjacent lines are equally spaced since the ^{82}Kr versus ^{84}Kr and ^{84}Kr versus ^{86}Kr mass differences are the same.

27.66. **IDENTIFY:** Apply conservation of energy to the acceleration of the ions and Newton's second law to their motion in the magnetic field.

SET UP: The singly ionized ions have $q = +e$. A ^{12}C ion has mass 12 u and a ^{14}C ion has mass 14 u, where $1\text{ u} = 1.66\times10^{-27}$ kg

EXECUTE: **(a)** During acceleration of the ions, $qV = \frac{1}{2}mv^2$ and $v = \sqrt{\frac{2qV}{m}}$. In the magnetic field,

$$R = \frac{mv}{qB} = \frac{m\sqrt{2qV/m}}{qB} \text{ and } m = \frac{qB^2R^2}{2V}.$$

(b) $V = \frac{qB^2R^2}{2m} = \frac{(1.60\times10^{-19}\text{ C})(0.150\text{ T})^2(0.500\text{ m})^2}{2(12)(1.66\times10^{-27}\text{ kg})} = 2.26\times10^4\text{ V}$

(c) The ions are separated by the differences in the diameters of their paths. $D = 2R = 2\sqrt{\frac{2Vm}{qB^2}}$, so

$$\Delta D = D_{14} - D_{12} = 2\sqrt{\frac{2Vm}{qB^2}}\bigg|_{14} - 2\sqrt{\frac{2Vm}{qB^2}}\bigg|_{12} = 2\sqrt{\frac{2V(1\text{ u})}{qB^2}}\left(\sqrt{14} - \sqrt{12}\right).$$

$\Delta D = 2\sqrt{\frac{2(2.26\times10^4\text{ V})(1.66\times10^{-27}\text{ kg})}{(1.6\times10^{-19}\text{ C})(0.150\text{ T})^2}}\left(\sqrt{14} - \sqrt{12}\right) = 8.01\times10^{-2}$ m. This is about 8 cm and is easily distinguishable.

EVALUATE: The speed of the ^{12}C ion is $v = \sqrt{\frac{2(1.60\times10^{-19}\text{ C})(2.26\times10^4\text{V})}{12(1.66\times10^{-27}\text{ kg})}} = 6.0\times10^5$ m/s. This is very fast, but well below the speed of light, so relativistic mechanics is not needed.

27.67. **IDENTIFY:** The force exerted by the magnetic field is given by Eq.(27.19). The net force on the wire must be zero.

SET UP: For the wire to remain at rest the force exerted on it by the magnetic field must have a component directed up the incline. To produce a force in this direction, the current in the wire must be directed from right to left in Figure 27.61 in the textbook. Or, viewing the wire from its left-hand end the directions are shown in Figure 27.67a.

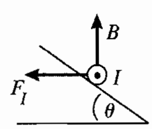


Figure 27.67a

The free-body diagram for the wire is given in Figure 27.67b.

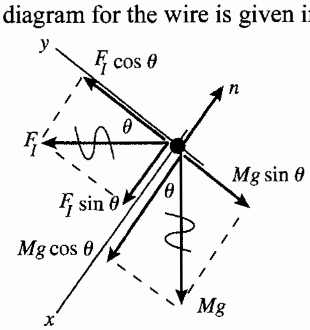


Figure 27.67b

EXECUTE: $\sum F_y = 0$

$F_I\cos\theta - Mg\sin\theta = 0$

$F_I = ILB\sin\phi$

$\phi = 90°$ since $\vec{B}$ is perpendicular to the current direction.

Thus $(ILB)\cos\theta - Mg\sin\theta = 0$ and $I = \frac{Mg\tan\theta}{LB}$

EVALUATE: The magnetic and gravitational forces are in perpendicular directions so their components parallel to the incline involve different trig functions. As the tilt angle θ increases there is a larger component of Mg down the incline and the component of F_I up the incline is smaller; I must increase with θ to compensate. As $\theta \to 0$, $I \to 0$ and as $\theta \to 90°, I \to \infty$.

27.68. **IDENTIFY:** The current in the bar is downward, so the magnetic force on it is vertically upwards. The net force on the bar is equal to the magnetic force minus the gravitational force, so Newton's second law gives the acceleration. The bar is in parallel with the $10.0\text{-}\Omega$ resistor, so we must use circuit analysis to find the initial current through it.

SET UP: First find the current. The equivalent resistance across the battery is 30.0 Ω, so the total current is 4.00 A, half of which goes through the bar. Applying Newton's second law to the bar gives $\sum F = ma = F_B - mg = iLB - mg$.

EXECUTE: Solving for the acceleration gives

$$a = \frac{iLB - mg}{m} = \frac{(2.0\text{ A})(1.50\text{ m})(1.60\text{ T}) - 3.00\text{ N}}{(3.00\text{ N}/9.80\text{ m/s}^2)} = 5.88\text{ m/s}^2.$$

The direction is upward.

EVALUATE: Once the bar is free of the conducting wires, its acceleration will become 9.8 m/s^2 downward since only gravity will be acting on it.

27.69. IDENTIFY: Calculate the acceleration of the ions when they first enter the field and assume this acceleration is constant. Apply conservation of energy to the acceleration of the ions by the potential difference.

SET UP: Assume $\vec{v} = v_x\hat{i}$ and neglect the y-component of $\vec{v}$ that is produced by the magnetic force.

EXECUTE: **(a)** $\frac{1}{2}mv_x^2 = qV$, so $v_x = \sqrt{\frac{2qV}{m}}$. Also, $a_y = \frac{qv_xB}{m}$ and $t = \frac{x}{v_x}$.

$$y = \tfrac{1}{2}a_yt^2 = \tfrac{1}{2}a_y\left(\frac{x}{v_x}\right)^2 = \frac{1}{2}\left(\frac{qv_xB}{m}\right)\left(\frac{x}{v_x}\right)^2 = \frac{1}{2}\left(\frac{qBx^2}{m}\right)\left(\frac{m}{2qV}\right)^{1/2} = Bx^2\left(\frac{q}{8mV}\right)^{1/2}.$$

(b) This can be used for isotope separation since the mass in the denominator leads to different locations for different isotopes.

EVALUATE: For $B = 0.1\text{ T}$, $v = 1\times10^5\text{ m/s}$, $q = +e$ and $m = 12\text{ u} = 2.0\times10^{-26}\text{ kg}$, $y = (4.0\text{ m}^{-1})x^2$. The approximation $y << x$ is valid as long as x is on the order of 10 cm or less.

27.70. IDENTIFY: Turning the charged loop creates a current, and the external magnetic field exerts a torque on that current.

SET UP: The current is $I = q/T = q/(1/f) = qf = q(\omega/2\pi) = q\omega/2\pi$. The torque is $\tau = \mu B\sin\phi$.

EXECUTE: In this case, $\phi = 90°$ and $\mu = AB$, giving $\tau = IAB$. Combining the results for the torque and current and using $A = \pi r^2$ gives $\tau = \left(\frac{q\omega}{2\pi}\right)\pi r^2B = \frac{1}{2}q\omega r^2B$

EVALUATE: Any moving charge is a current, so turning the loop creates a current causing a magnetic force.

27.71. IDENTIFY: $R = \frac{mv}{|q|B}$.

SET UP: After completing one semicircle the separation between the ions is the difference in the diameters of their paths, or $2(R_{13} - R_{12})$. A singly ionized ion has charge $+e$.

EXECUTE: **(a)** $B = \frac{mv}{|q|R} = \frac{(1.99\times10^{-26}\text{ kg})(8.50\times10^3\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.125\text{ m})} = 8.46\times10^{-3}\text{ T}$.

(b) The only difference between the two isotopes is their masses. $\frac{R}{m} = \frac{v}{|q|B} = \text{constant}$ and $\frac{R_{12}}{m_{12}} = \frac{R_{13}}{m_{13}}$.

$R_{13} = R_{12}\left(\frac{m_{13}}{m_{12}}\right) = (12.5\text{ cm})\left(\frac{2.16\times10^{-26}\text{ kg}}{1.99\times10^{-26}\text{ kg}}\right) = 13.6\text{ cm}$. The diameter is 27.2 cm.

(c) The separation is $2(R_{13} - R_{12}) = 2(13.6\text{ cm} - 12.5\text{ cm}) = 2.2\text{ cm}$. This distance can be easily observed.

EVALUATE: Decreasing the magnetic field increases the separation between the two isotopes at the detector.

27.72. IDENTIFY: The force exerted by the magnetic field is $F = ILB\sin\phi$. $a = F/m$ and is constant. Apply a constant acceleration equation to relate v and d.

SET UP: $\phi = 90°$. The direction of $\vec{F}$ is given by the right-hand rule.

EXECUTE: **(a)** $F = ILB$, to the right.

(b) $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v^2 = 2ad$ and $d = \frac{v^2}{2a} = \frac{v^2m}{2ILB}$.

(c) $d = \frac{(1.12\times10^4\text{ m/s})^2(25\text{ kg})}{2(2000\text{ A})(0.50\text{ m})(0.50\text{ T})} = 3.14\times10^6\text{ m} = 3140\text{ km}$

EVALUATE: $a = \frac{ILB}{m} = \frac{(2.0\times10^3\text{ A})(0.50\text{ m})(0.50\text{ T})}{25\text{ kg}} = 20\text{ m/s}^2$. The acceleration due to gravity is not negligible.

27.73. **IDENTIFY:** Apply $F = IlB\sin\phi$ to calculate the force on each segment of the wire that is in the magnetic field. The net force is the vector sum of the forces on each segment.

SET UP: The direction of the magnetic force on each current segment in the field is shown in Figure 27.73. By symmetry, $F_a = F_b$. $\vec{F}_a$ and $\vec{F}_b$ are in opposite directions so their vector sum is zero. The net force equals F_c. For F_c, $\phi = 90°$ and $l = 0.450$ m.

EXECUTE: $F_c = IlB = (6.00\text{ A})(0.450\text{ m})(0.666\text{ T}) = 1.80\text{ N}$. The net force is 1.80 N, directed to the left.

EVALUATE: The shape of the region of uniform field doesn't matter, as long as all of segment c is in the field and as long as the lengths of the portions of segments a and b that are in the field are the same.

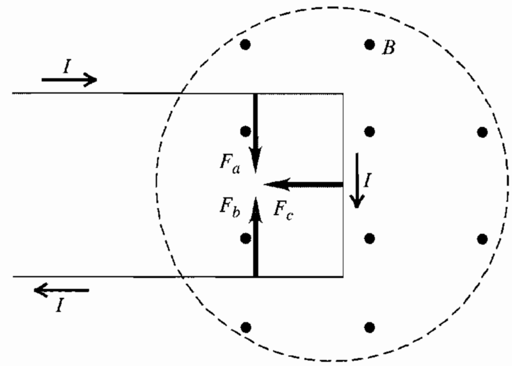


Figure 27.73

27.74. **IDENTIFY:** Apply $\vec{F} = I\vec{l} \times \vec{B}$.

SET UP: $\vec{l} = l\hat{k}$

EXECUTE: **(a)** $\vec{F} = I(l\hat{k}) \times \vec{B} = Il\left[(-B_y)\hat{i} + (B_x)\hat{j}\right]$. This gives

$F_x = -IlB_y = -(9.00\text{ A})\,(0.250\text{ m})(-0.985\text{ T}) = 2.22\text{ N}$ and $F_y = IlB_x = (9.00\text{ A})(0.250\text{ m})(-0.242\text{ T}) = -0.545\text{ N}$..

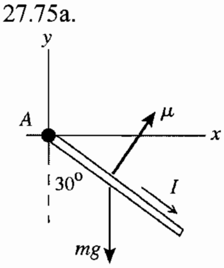
$F_z = 0$, since the wire is in the z-direction.

(b) $F = \sqrt{F_x^2 + F_y^2} = \sqrt{(2.22\text{ N})^2 + (0.545\text{ N})^2} = 2.29\text{ N}$.

EVALUATE: $\vec{F}$ must be perpendicular to the current direction, so $\vec{F}$ has no z component.

27.75. **IDENTIFY:** For the loop to be in equilibrium the net torque on it must be zero. Use Eq.(27.26) to calculate the torque due to the magnetic field and use Eq.(10.3) for the torque due to the gravity force.

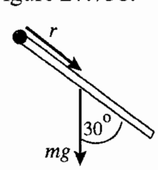
SET UP: See Figure 27.75a.



Use $\sum \tau_A = 0$, where point A is at the origin.

Figure 27.75a

EXECUTE: See Figure 27.75b.



$\tau_{mg} = mgr\sin\phi = mg(0.400\text{ m})\sin 30.0°$

The torque is clockwise; $\vec{\tau}_{mg}$ is directed into the paper.

Figure 27.75b

For the loop to be in equilibrium the torque due to $\vec{B}$ must be counterclockwise (opposite to $\vec{\tau}_{mg}$) and it must be that $\tau_B = \tau_{mg}$. See Figure 27.75c.

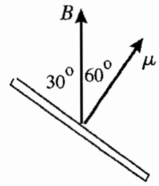


Figure 27.75c

$\vec{\tau}_B = \vec{\mu}\times\vec{B}$. For this torque to be counterclockwise ($\vec{\tau}_B$ directed out of the paper), $\vec{B}$ must be in the $+y$-direction.

$\tau_B = \mu B\sin\phi = IAB\sin 60.0°$

$\tau_B = \tau_{mg}$ gives $IAB\sin 60.0° = mg(0.0400\text{ m})\sin 30.0°$

$m = (0.15\text{ g/cm})2(8.00\text{ cm} + 6.00\text{ cm}) = 4.2\text{ g} = 4.2\times10^{-3}\text{ kg}$

$A = (0.0800\text{ m})(0.0600\text{ m}) = 4.80\times10^{-3}\text{ m}^2$

$$B = \frac{mg(0.0400\text{ m})(\sin 30.0°)}{IA\sin 60.0°}$$

$$B = \frac{(4.2\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)(0.0400\text{ m})\sin 30.0°}{(8.2\text{ A})(4.80\times10^{-3}\text{ m}^2)\sin 60.0°} = 0.024\text{ T}$$

EVALUATE: As the loop swings up the torque due to $\vec{B}$ decreases to zero and the torque due to mg increases from zero, so there must be an orientation of the loop where the net torque is zero.

27.76. **IDENTIFY:** The torque exerted by the magnetic field is $\vec{\tau} = \vec{\mu}\times\vec{B}$. The torque required to hold the loop in place is $-\vec{\tau}$.

SET UP: $\mu = IA$. $\vec{\mu}$ is normal to the plane of the loop, with a direction given by the right-hand rule that is illustrated in Figure 27.32 in the textbook. $\tau = IAB\sin\phi$, where ϕ is the angle between the normal to the loop and the direction of $\vec{B}$.

EXECUTE: **(a)** $\tau = IAB\sin 60° = (15.0\text{ A})(0.060\text{ m})(0.080\text{ m})(0.48\text{ T})\sin 60° = 0.030\text{ N}\cdot\text{m}$, in the $-\hat{j}$ direction. To keep the loop in place, you must provide a torque in the $+\hat{j}$ direction.

(b) $\tau = IAB\sin 30° = (15.0\text{ A})(0.60\text{ m})(0.080\text{ m})(0.48\text{ T})\sin 30° = 0.017\text{ N}\cdot\text{m}$, in the $+\hat{j}$ direction. You must provide a torque in the $-\hat{j}$ direction to keep the loop in place.

EVALUATE: **(c)** If the loop was pivoted through its center, then there would be a torque on both sides of the loop parallel to the rotation axis. However, the lever arm is only half as large, so the total torque in each case is identical to the values found in parts (a) and (b).

27.77. **IDENTIFY:** Use Eq.(27.20) to calculate the force and then the torque on each small section of the rod and integrate to find the total magnetic torque. At equilibrium the torques from the spring force and from the magnetic force cancel. The spring force depends on the amount x the spring is stretched and then $U = \frac{1}{2}kx^2$ gives the energy stored in the spring.

(a) SET UP:

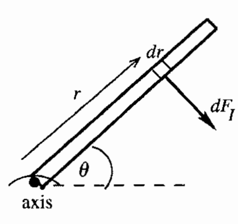


Figure 27.77

Divide the rod into infinitesimal sections of length dr, as shown in Figure 27.77.

EXECUTE: The magnetic force on this section is $dF_I = IBdr$ and is perpendicular to the rod. The torque $d\tau$ due to the force on this section is $d\tau = rdF_I = IBr\,dr$. The total torque is $\int d\tau = IB\int_0^l r\,dr = \frac{1}{2}Il^2B = 0.0442\text{ N}\cdot\text{m}$, clockwise.

(b) SET UP: F_I produces a clockwise torque so the spring force must produce a counterclockwise torque. The spring force must be to the left; the spring is stretched.

EXECUTE: Find x, the amount the spring is stretched:

$\sum\tau = 0$, axis at hinge, counterclockwise torques positive

$(kx)l\sin 53° - \frac{1}{2}Il^2B = 0$

$$x = \frac{IlB}{2k\sin 53.0°} = \frac{(6.50\ \text{A})(0.200\ \text{m})(0.340\ \text{T})}{2(4.80\ \text{N/m})\sin 53.0°} = 0.05765\ \text{m}$$

(c) $U = \frac{1}{2}kx^2 = 7.98\times10^{-3}\ \text{J}$

EVALUATE: The magnetic torque calculated in part (a) is the same torque calculated from a force diagram in which the total magnetic force $F_I = IlB$ acts at the center of the rod. We didn't include a gravity torque since the problem said the rod had negligible mass.

27.78. **IDENTIFY:** Apply $\vec{F} = I\vec{l}\times\vec{B}$ to calculate the force on each side of the loop.

SET UP: The net force is the vector sum of the forces on each side of the loop.

EXECUTE: **(a)** $F_{PQ} = (5.00\ \text{A})(0.600\ \text{m})(3.00\ \text{T})\sin(0°) = 0\ \text{N}$.

$F_{RP} = (5.00\ \text{A})\ (0.800\ \text{m})\ (3.00\ \text{T})\sin(90°) = 12.0\ \text{N}$, into the page.

$F_{QR} = (5.00\ \text{A})(1.00\ \text{m})(3.00\ \text{T})(0.800/1.00) = 12.0\ \text{N}$, out of the page.

(b) The net force on the triangular loop of wire is zero.

(c) For calculating torque on a straight wire we can assume that the force on a wire is applied at the wire's center. Also, note that we are finding the torque with respect to the *PR*-axis (not about a point), and consequently the lever arm will be the distance from the wire's center to the *x*-axis. $\tau = rF\sin\phi$ gives $\tau_{PQ} = r(0\ \text{N}) = 0$,

$\tau_{RP} = (0\ \text{m})F\sin\phi = 0$ and $\tau_{QR} = (0.300\ \text{m})(12.0\ \text{N})\sin(90°) = 3.60\ \text{N}\cdot\text{m}$. The net torque is $3.60\ \text{N}\cdot\text{m}$.

(d) According to Eq.(27.28), $\tau = NIAB\sin\phi = (1)(5.00\ \text{A})(\frac{1}{2})(0.600\ \text{m})(0.800\ \text{m})(3.00\ \text{T})\sin(90°) = 3.60\ \text{N}\cdot\text{m}$, which agrees with part (c).

(e) Since F_{QR} is out of the page and since this is the force that produces the net torque, the point Q will be rotated out of the plane of the figure.

EVALUATE: In the expression $\tau = NIAB\sin\phi$, ϕ is the angle between the plane of the loop and the direction of $\vec{B}$. In this problem, $\phi = 90°$.

27.79. **IDENTIFY:** Use Eq.(27.20) to calculate the force on a short segment of the coil and integrate over the entire coil to find the total force.

SET UP: See Figure 27.79a.

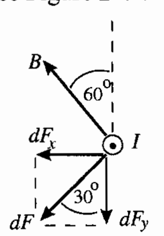


Figure 27.79a

Consider the force $d\vec{F}$ on a short segment dl at the left-hand side of the coil, as viewed in Figure 27.69 in the textbook. The current at this point is directed out of the page. $d\vec{F}$ is perpendicular both to $\vec{B}$ and to the direction of I.

See Figure 27.79b.

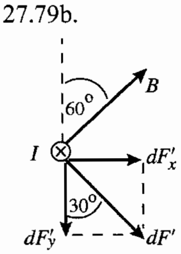


Figure 27.79b

Consider also the force $d\vec{F}'$ on a short segment on the opposite side of the coil, at the right-hand side of the coil in Figure 27.69 in the textbook. The current at this point is directed into the page.

The two sketches show that the *x*-components cancel and that the *y*-components add. This is true for all pairs of short segments on opposite sides of the coil. The net magnetic force on the coil is in the *y*-direction and its magnitude is given by $F = \int dF_y$.

EXECUTE: $dF = Idl\,B\sin\phi$. But $\vec{B}$ is perpendicular to the current direction so $\phi = 90°$.

$dF_y = dF\cos 30.0 = IB\cos 30.0° dl$

$F = \int dF_y = IB\cos 30.0° \int dl$

But $\int dl = N(2\pi r)$, the total length of wire in the coil.

$F = IB\cos 30.0° N(2\pi r) = (0.950\text{ A})(0.220\text{ T})(\cos 30.0°)(50)2\pi(0.0078\text{ m}) = 0.444\text{ N}$ and $\vec{F} = -(0.444\text{ N})\hat{j}$

EVALUATE: The magnetic field makes a constant angle with the plane of the coil but has a different direction at different points around the circumference of the coil so is not uniform. The net force is proportional to the magnitude of the current and reverses direction when the current reverses direction.

27.80. **IDENTIFY:** Conservation of energy relates the accelerating potential difference V to the final speed of the ions. In the magnetic field region the ions travel in an arc of a circle that has radius $R = \dfrac{mv}{|q|B}$.

SET UP: The quarter-circle paths of the two ions are shown in Figure 27.80. The separation at the detector is $\Delta r = R_{18} - R_{16}$. Each ion has charge $q = +e$.

EXECUTE: (a) Conservation of energy gives $|q|V = \frac{1}{2}mv^2$ and $v = \sqrt{\dfrac{2|q|V}{m}}$. $R = \dfrac{m}{|q|B}\sqrt{\dfrac{2|q|V}{m}} = \dfrac{\sqrt{2|q|mV}}{|q|B}$.

$|q| = e$ for each ion. $\Delta r = R_{18} - R_{16} = \dfrac{\sqrt{2eV}}{eB}(\sqrt{m_{18}} - \sqrt{m_{16}})$.

(b) $V = \dfrac{(\Delta reB)^2}{2e\left(\sqrt{m_{18}} - \sqrt{m_{16}}\right)^2} = \dfrac{e(\Delta r)^2 B^2}{2\left(\sqrt{m_{18}} - \sqrt{m_{16}}\right)^2} = \dfrac{(1.60\times10^{-19}\text{ C})(4.00\times10^{-2}\text{ m})^2(0.050\text{ T})^2}{2\left(\sqrt{2.99\times10^{-26}\text{ kg}} - \sqrt{2.66\times10^{-26}\text{ kg}}\right)^2}$

$V = 3.32\times10^3\text{ V}$.

EVALUATE: The speed of the ^{16}O ion after it has been accelerated through a potential difference of $V = 3.32\times10^3\text{ V}$ is $2.00\times10^5\text{ m/s}$. Increasing the accelerating voltage increases the separation of the two isotopes at the detector. But it does this by increasing the radius of the path for each ion, and this increases the required size of the magnetic field region.

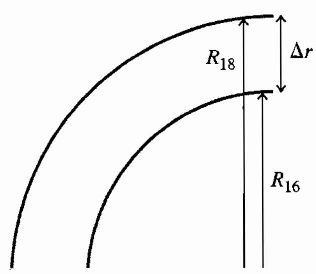


Figure 27.80

27.81. **IDENTIFY:** Apply $d\vec{F} = Id\vec{l} \times \vec{B}$ to each side of the loop.

SET UP: For each side of the loop, $d\vec{l}$ is parallel to that side of the loop and is in the direction of I. Since the loop is in the xy-plane, $z = 0$ at the loop and $B_y = 0$ at the loop.

EXECUTE: (a) The magnetic field lines in the yz-plane are sketched in Figure 27.81.

(b) Side 1, that runs from (0,0) to (0,L): $\vec{F} = \int_0^L Id\vec{l} \times \vec{B} = I\int_0^L \dfrac{B_0 y\,dy}{L}\hat{i} = \frac{1}{2}B_0 LI\hat{i}.$

Side 2, that runs from (0,L) to (L,L): $\vec{F} = \int_{0,y=L}^{L} Id\vec{l} \times \vec{B} = I\int_{0,y=L}^{L} \dfrac{B_0 y\,dx}{L}\hat{j} = -IB_0L\hat{j}$.

Side 3, that runs from (L,L) to (L,0): $\vec{F} = \int_{L,x=L}^{0} Id\vec{l} \times \vec{B} = I\int_{L,x=L}^{0} \dfrac{B_0 y\,dy}{L}(-\hat{i}) = -\frac{1}{2}IB_0L\hat{i}$.

Side 4, that runs from (L,0) to (0,0): $\vec{F} = \int_{L,y=0}^{0} Id\vec{l} \times \vec{B} = I\int_{L,y=0}^{0} \dfrac{B_0 y\,dx}{L}\hat{j} = 0.$

(c) The sum of all forces is $\vec{F}_{\text{total}} = -IB_0L\hat{j}$.

EVALUATE: The net force on sides 1 and 3 is zero. The force on side 4 is zero, since $y = 0$ and $z = 0$ at that side and therefore $B = 0$ there. The net force on the loop equals the force on side 2.

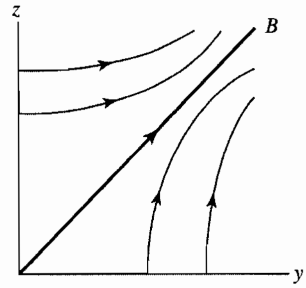


Figure 27.81

27.82. **IDENTIFY:** Apply $d\vec{F} = Id\vec{l} \times \vec{B}$ to each side of the loop. $\vec{\tau} = \vec{r} \times \vec{F}$.

SET UP: For each side of the loop, $d\vec{l}$ is parallel to that side of the loop and is in the direction of I.

EXECUTE: **(a)** The magnetic field lines in the xy-plane are sketched in Figure 27.82.

(b) Side 1, that runs from (0,0) to (0,L): $\vec{F} = \int_0^L Id\vec{l} \times \vec{B} = I\int_0^L \frac{B_0 y\, dy}{L}(-\hat{k}) = -\frac{1}{2}B_0 LI\hat{k}$.

Side 2, that runs from (0,L) to (L,L): $\vec{F} = \int_0^L Id\vec{l} \times \vec{B} = I\int_0^L \frac{B_0 x\, dx}{L}\hat{k} = \frac{1}{2}IB_0 L\hat{k}.$

Side 3, that runs from (L,L) to (L,0): $\vec{F} = \int_0^L Id\vec{l} \times \vec{B} = I\int_0^L \frac{B_0 y dy}{L}\hat{k} = +\frac{1}{2}IB_0 L\hat{k}$

Side 4, that runs from (L,0) to (0,0): $\vec{F} = \int_0^L Id\vec{l} \times \vec{B} = I\int_0^L \frac{B_0 x dx}{L}(-\hat{k}) = -\frac{1}{2}IB_0 L\hat{k}.$

(c) If free to rotate about the x-axis, the torques due to the forces on sides 1 and 3 cancel and the torque due to the forces on side 4 is zero. For side 2, $\vec{r} = L\hat{j}$. Therefore, $\vec{\tau} = \vec{r} \times \vec{F} = \frac{IB_0 L^2}{2}\hat{i} = \frac{1}{2}IAB_0\hat{i}$.

(d) If free to rotate about the y-axis, the torques due to the forces on sides 2 and 4 cancel and the torque due to the forces on side 1 is zero. For side 3, $\vec{r} = L\hat{i}$. Therefore, $\vec{\tau} = \vec{r} \times \vec{F} = -\frac{IB_0 L^2}{2}\hat{j} = -\frac{1}{2}IAB_0\hat{j}$.

EVALUATE: **(e)** The equation for the torque $\vec{\tau} = \vec{\mu} \times \vec{B}$ is not appropriate, since the magnetic field is not constant.

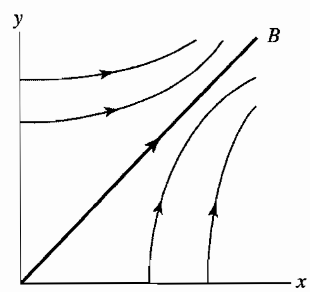


Figure 27.82

27.83. **IDENTIFY:** While the ends of the wire are in contact with the mercury and current flows in the wire, the magnetic field exerts an upward force and the wire has an upward acceleration. After the ends leave the mercury the electrical connection is broken and the wire is in free-fall.

(a) SET UP: After the wire leaves the mercury its acceleration is g, downward. The wire travels upward a total distance of 0.350 m from its initial position. Its ends lose contact with the mercury after the wire has traveled 0.025 m, so the wire travels upward 0.325 m after it leaves the mercury. Consider the motion of the wire after it leaves the mercury. Take $+y$ to be upward and take the origin at the position of the wire as it leaves the mercury.

$a_y = -9.80\ \text{m/s}^2$, $y - y_0 = +0.325$ m, $v_y = 0$ (at maximum height), $v_{0y} = ?$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

EXECUTE: $v_{0y} = \sqrt{-2a_y(y - y_0)} = \sqrt{-2(-9.80\ \text{m/s}^2)(0.325\ \text{m})} = 2.52$ m/s

(b) Set Up: Now consider the motion of the wire while it is in contact with the mercury. Take $+y$ to be upward and the origin at the initial position of the wire. Calculate the acceleration: $y-y_0=+0.025$ m, $v_{0y}=0$ (starts from rest), $v_y=+2.52$ m/s (from part (a)), $a_y=?$

$$v_y^2=v_{0y}^2+2a_y(y-y_0)$$

Execute: $a_y=\dfrac{v_y^2}{2(y-y_0)}=\dfrac{(2.52\text{ m/s})^2}{2(0.025\text{ m})}=127\text{ m/s}^2$

Set Up: The free-body diagram for the wire is given in Figure 27.83.

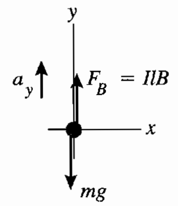


Figure 27.83

Execute: $\sum F_y=ma_y$

$F_B-mg=ma_y$

$IlB=m(g+a_y)$

$I=\dfrac{m(g+a_y)}{lB}$

l is the length of the horizontal section of the wire; $l=0.150$ m

$$I=\frac{(5.40\times10^{-5}\text{ kg})(9.80\text{ m/s}^2+127\text{ m/s}^2)}{(0.150\text{ m})(0.00650\text{ T})}=7.58\text{ A}$$

(c) Identify and **Set Up:** Use Ohm's law.

Execute: $V=IR$ so $R=\dfrac{V}{I}=\dfrac{1.50\text{ V}}{7.58\text{ A}}=0.198\ \Omega$

Evaluate: The current is large and the magnetic force provides a large upward acceleration. During this upward acceleration the wire moves a much shorter distance as it gains speed than the distance is moves while in free-fall with a much smaller acceleration, as it loses the speed it gained. The large current means the resistance of the wire must be small.

27.84. **Identify** and **Set Up:** Follow the procedures specified in the problem.

Execute: **(a)** $d\vec{l}=dl\hat{t}$, where $\hat{t}$ is a unit vector in the tangential direction. $d\vec{l}=Rd\theta\left[-\sin\theta\hat{i}+\cos\theta\hat{j}\right]$. Note that this implies that when $\theta=0$, the line element points in the $+y$-direction, and when the angle is $90°$, the line element points in the $-x$-direction. This is in agreement with the diagram.

$d\vec{F}=Id\vec{l}\times\vec{B}=IRd\theta\left[-\sin\theta\hat{i}+\cos\theta\hat{j}\right]\times(B_x\hat{i})=IB_xRd\theta\left[-\cos\theta\hat{k}\right]$.

(b) $\vec{F}=\int_0^{2\pi}-\cos\theta IB_xR\,d\theta\hat{k}=-IB_xR\int_0^{2\pi}\cos\theta d\theta\hat{k}=0.$

(c) $d\vec{\tau}=\vec{r}\times d\vec{F}=R(\cos\theta\hat{i}+\sin\theta\hat{j})\times(IB_xR\,d\theta[-\cos\theta\hat{k}])=-R^2IB_xd\theta(\sin\theta\cos\theta\hat{i}-\cos^2\theta\hat{j})$

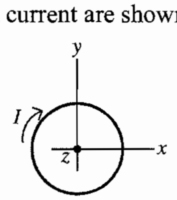
(d) $\vec{\tau}=\int d\vec{\tau}=-R^2IB_x\left(\int_0^{2\pi}\sin\theta\cos\theta d\theta\hat{i}-\int_0^{2\pi}\cos^2\theta d\theta\hat{j}\right)=IR^2B_x\left(\dfrac{\theta}{2}+\dfrac{\sin2\theta}{4}\right)_0^{2\pi}\hat{j}$. $\vec{\tau}=IR^2B_x\pi\hat{j}=I\pi R^2B_x\hat{j}=IA\hat{k}\times B_x\hat{i}$

and $\vec{\tau}=\vec{\mu}\times\vec{B}$.

Evaluate: Section 27.7 of the textbook derived $\vec{\tau}=\vec{\mu}\times\vec{B}$ for the case of a rectangular coil. This problem shows that the same result also applies to a circular coil.

27.85. **(a) Identify:** Use Eq.(27.27) to relate U, μ and $\vec{B}$ and use Eq.(27.26) to relate $\vec{\tau}$, $\vec{\mu}$ and $\vec{B}$. We also know that $B_0^2=B_x^2+B_y^2+B_z^2$. This gives three equations for the three components of $\vec{B}$.

Set Up: The loop and current are shown in Figure 27.85.



$\vec{\mu}$ is into the plane of the paper, in the $-z$-direction

Figure 27.85

$\vec{\mu}=-\mu\hat{k}=-IA\hat{k}$

(b) Execute: $\vec{\tau}=D(+4\hat{i}-3\hat{j})$, where $D>0$.

$\vec{\mu}=-IA\hat{k}$, $\vec{B}=B_x\hat{i}+B_y\hat{j}+B_y\hat{k}$

$\vec{\tau} = \vec{\mu}\times\vec{B} = (-IA)(B_x\hat{k}\times\hat{i} + B_y\hat{k}\times\hat{j} + B_z\hat{k}\times\hat{k}) = IAB_y\hat{i} - IAB_x\hat{j}$

Compare this to the expression given for $\vec{\tau}$: $IAB_y = 4D$ so $B_y = 4D/IA$ and $-IAB_x = -3D$ so $B_x = 3D/IA$

B_z doesn't contribute to the torque since $\vec{\mu}$ is along the z-direction. But $B = B_0$ and $B_x^2 + B_y^2 + B_z^2 = B_0^2$; with $B_0 = 13D/IA$. Thus $B_z = \pm\sqrt{B_0^2 - B_x^2 - B_y^2} = \pm(D/IA)\sqrt{169-9-16} = \pm 12(D/IA)$

That $U = -\vec{\mu}\cdot\vec{B}$ is negative determines the sign of B_z: $U = -\vec{\mu}\cdot\vec{B} = -(-IA\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) = +IAB_z$

So U negative says that B_z is negative, and thus $B_z = -12D/IA$.

EVALUATE: $\vec{\mu}$ is along the z-axis so only B_x and B_y contribute to the torque. B_x produces a y-component of $\vec{\tau}$ and B_y produces an x-component of $\vec{\tau}$. Only B_z affects U, and U is negative when $\vec{\mu}$ and $\vec{B}_z$ are parallel.

27.86. IDENTIFY: $I = \dfrac{\Delta q}{\Delta t}$ and $\mu = IA$.

SET UP: The direction of $\vec{\mu}$ is given by the right-hand rule that is illustrated in Figure 27.32 in the textbook. I is in the direction of flow of positive charge and opposite to the direction of flow of negative charge.

EXECUTE: (a) $I_u = \dfrac{dq}{dt} = \dfrac{\Delta q}{\Delta t} = \dfrac{q_u v}{2\pi r} = \dfrac{ev}{3\pi r}$.

(b) $\mu_u = I_u A = \dfrac{ev}{3\pi r}\pi r^2 = \dfrac{evr}{3}$.

(c) Since there are two down quarks, each of half the charge of the up quark, $\mu_d = \mu_u = \dfrac{evr}{3}$. Therefore, $\mu_{\text{total}} = \dfrac{2evr}{3}$.

(d) $v = \dfrac{3\mu}{2er} = \dfrac{3(9.66\times10^{-27}\ \text{A}\cdot\text{m}^2)}{2(1.60\times10^{-19}\ \text{C})(1.20\times10^{-15}\text{m})} = 7.55\times10^{7}\ \text{m/s}$.

EVALUATE: The speed calculated in part (d) is 25% of the speed of light.

27.87. IDENTIFY: Eq.(27.8) says that the magnetic field through any closed surface is zero.

SET UP: The cylindrical Gaussian surface has its top at $z = L$ and its bottom at $z = 0$. The rest of the surface is the curved portion of the cylinder and has radius r and length L. $B = 0$ at the bottom of the surface, since $z = 0$ there.

EXECUTE: (a) $\vec{B}\cdot d\vec{A} = \int_{\text{top}} B_z dA + \int_{\text{curved}} B_r dA = \int_{\text{top}} (\beta L) dA + \int_{\text{curved}} B_r dA = 0$. This gives $0 = \beta L\pi r^2 + B_r 2\pi r L$, and

$B_r(r) = -\dfrac{\beta r}{2}$.

(b) The two diagrams in Figure 27.87 show views of the field lines from the top and side of the Gaussian surface.

EVALUATE: Only a portion of each field line is shown; the field lines are closed loops.

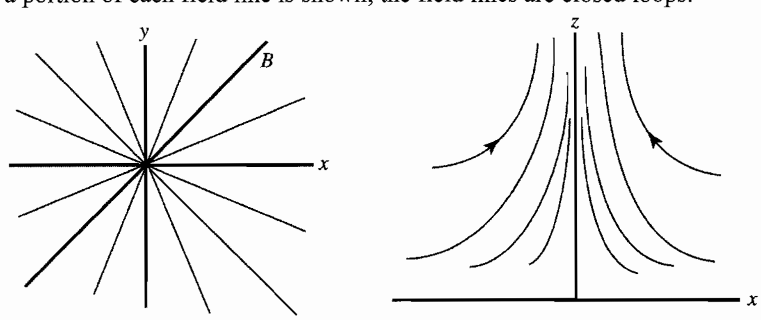


Figure 27.87

27.88. IDENTIFY: $U = -\vec{\mu}\cdot\vec{B}$. In part (b) apply conservation of energy.

SET UP: The kinetic energy of the rotating ring is $K = \frac{1}{2}I\omega^2$.

EXECUTE: (a) $\Delta U = -(\vec{\mu}_f\cdot\vec{B} - \vec{\mu}_i\cdot\vec{B}) = -(\vec{\mu}_f - \vec{\mu}_i)\cdot\vec{B} = \left[-\mu(-\hat{k} - (-0.8\hat{i} + 0.6\hat{j}))\right]\cdot\left[B_0(12\hat{i} + 3\hat{j} - 4\hat{k})\right]$.

$\Delta U = IAB_0[(-0.8)(+12) + (0.6)(+3) + (+1)(-4)] = (12.5\ \text{A})(4.45\times10^{-4}\ \text{m}^2)(0.0115\ \text{T})(-11.8)$.

$\Delta U = -7.55\times10^{-4}\text{J}$.

(b) $\Delta K = \dfrac{1}{2}I\omega^2$. $\omega = \sqrt{\dfrac{2\Delta K}{I}} = \sqrt{\dfrac{2(7.55\times10^{-4}\ \text{J})}{8.50\times10^{-7}\ \text{kg}\cdot\text{m}^2}} = 42.1\ \text{rad/s}$.

EVALUATE: The potential energy of the ring decreases and its kinetic energy increases.

27.89. **IDENTIFY** and **SET UP:** In the magnetic field, $R = \dfrac{mv}{qB}$. Once the particle exits the field it travels in a straight line. Throughout the motion the speed of the particle is constant.

EXECUTE: **(a)** $R = \dfrac{mv}{qB} = \dfrac{(3.20\times10^{-11}\text{ kg})(1.45\times10^{5}\text{ m/s})}{(2.15\times10^{-6}\text{ C})(0.420\text{ T})} = 5.14\text{ m}.$

(b) See Figure 27.89. The distance along the curve, d, is given by $d = R\theta$. $\sin\theta = \dfrac{0.25\text{ m}}{5.14\text{ m}}$, so

$\theta = 2.79° = 0.0486$ rad. $d = R\theta = (5.14\text{ m})(0.0486\text{ rad}) = 0.25$ m. And $t = \dfrac{d}{v} = \dfrac{0.25\text{ m}}{1.45\times10^{5}\text{ m/s}} = 1.72\times10^{-6}\text{s}.$

(c) $\Delta x_1 = d\tan(\theta/2) = (0.25\text{ m})\tan(2.79°/2) = 6.08\times10^{-3}$ m.

(d) $\Delta x = \Delta x_1 + \Delta x_2$, where Δx_2 is the horizontal displacement of the particle from where it exits the field region to where it hits the wall. $\Delta x_2 = (0.50\text{ m})\tan 2.79° = 0.0244$ m. Therefore, $\Delta x = 6.08\times10^{-3}\text{ m} + 0.0244\text{ m} = 0.0305$ m.

EVALUATE: d is much less than R, so the horizontal deflection of the particle is much smaller than the distance it travels in the y-direction.

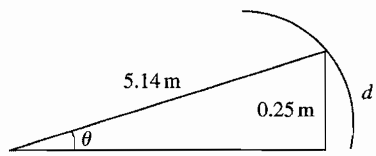


Figure 27.89

27.90. **IDENTIFY:** The current direction is perpendicular to $\vec{B}$, so $F = IlB$. If the liquid doesn't flow, a force $(\Delta p)A$ from the pressure difference must oppose F.

SET UP: $J = I/A$, where $A = hw$.

EXECUTE: **(a)** $\Delta p = F/A = IlB/A = JlB.$

(b) $J = \dfrac{\Delta p}{lB} = \dfrac{(1.00\text{ atm})(1.013\times10^{5}\text{ Pa/atm})}{(0.0350\text{ m})(2.20\text{ T})} = 1.32\times10^{6}\text{A/m}^2.$

EVALUATE: A current of 1 A in a wire with diameter 1 mm corresponds to a current density of $J = 1.3\times10^{6}\text{ A/m}^2$, so the current density calculated in part (c) is a typical value for circuits.

27.91. **IDENTIFY:** The electric and magnetic fields exert forces on the moving charge. The work done by the electric field equals the change in kinetic energy. At the top point, $a_y = \dfrac{v^2}{R}$ and this acceleration must correspond to the net force.

SET UP: The electric field is uniform so the work it does for a displacement y in the y-direction is $W = Fy = qEy$. At the top point, $\vec{F}_B$ is in the $-y$-direction and $\vec{F}_E$ is in the $+y$-direction.

EXECUTE: **(a)** The maximum speed occurs at the top of the cycloidal path, and hence the radius of curvature is greatest there. Once the motion is beyond the top, the particle is being slowed by the electric field. As it returns to $y = 0$, the speed decreases, leading to a smaller magnetic force, until the particle stops completely. Then the electric field again provides the acceleration in the y-direction of the particle, leading to the repeated motion.

(b) $W = qEy = \dfrac{1}{2}mv^2$ and $v = \sqrt{\dfrac{2qEy}{m}}.$

(c) At the top, $F_y = qE - qvB = -\dfrac{mv^2}{R} = -\dfrac{m}{2y}\dfrac{2qEy}{m} = -qE.$ $2qE = qvB$ and $v = \dfrac{2E}{B}.$

EVALUATE: The speed at the top depends on B because B determines the y-displacement and the work done by the electric force depends on the y-displacement.